1 Search and Marriage

- Each adult lives two periods.
- Each individual has has a reservation utility per period of remaining single, s.
- Assume that the ages of the participants are observed by everyone.
- Adults are willing only to marry others of the same age.
- When a young male and a young female meet in the first period, they draw an indiosyncratic match value θ from the cumulative distribution $F(\theta)$. If they marry, each individual gets the match value θ for every period of marriage.

- If individuals decide not to marry, they can return to the marriage market in the second period to draw another match value.

- Before drawing θ, a single individual at the beginning of the second period will enjoy an expected utility of

$$E \max\{s, \theta\} = sF(s) + (1 - F(s))E(\theta | \theta \geq s)$$

E is the expectations operator.

- Note that $E \max\{s, \theta\} > s$.

Consider a young female's decision problem.

Let her draw θ_1 in the first period. Her present value from marriage in the first period without discounting is:

$$V_m(\theta_1) = \theta_1 + \theta_1 \tag{1}$$

Her expected present value from remaining single is:

$$V_s = s + E \max\{s, \theta\} \tag{2}$$

Since $E \max\{s, \theta\} > s$, $V_s > 2s$.

Her decision whether to marry or remain single depends on:

$$\begin{aligned} V(\theta_1) &= \max\{V_m(\theta_1), V_s\} \\ &= \max\{2\theta_1, V_s\} \end{aligned}$$

$\frac{V_s}{2}$ is her reservation match value.

- Since $\frac{V_s}{2} > s$, she will reject matches, θ_1, in which $\frac{V_s}{2} > \theta_1 \geq s$.

- The probability that she will marry in the first period is $1 - F(\frac{V_s}{2})$.

- If she is single at the beginning of the second period, the probability that she will marry is $1 - F(s) > 1 - F(\frac{V_s}{2})$

2 Divorce

- Let young married adults divorce when they are old.

- Let a young married couple experience θ_1 in the first period.

- In the second period, they will want to divorce if

$$\theta_1 < E \max\{s, \theta\}$$

- A young married woman with match value θ_1 will receive in the second period:

$$V_2(\theta_1) = \max[\theta_1, E \max\{s, \theta\}]$$

- $V_2(\theta_1) \geq E \max\{s, \theta_2\}$

- Thus if $\theta_1 > s$, the individual should marry in the first period.

- We will also observe positive assortative matching by age in marriage.

- Divorce is strictly welfare improving in this society.

- More individuals will marry in a sociey with divorce than without.

- There are some predictable divorce.

3 Arrival of new information

Let θ_1 be the match value in the first period.

- Let θ_2 be the match value in the second period where

$$\begin{aligned} \theta_2 &= \theta_1 + \lambda + \omega \\ E\theta_2 &= \theta_1 + \lambda \end{aligned} \tag{3}$$

- ω is drawn from a density $g(\omega)$ and distribution function $G(\omega)$ and has a mean of 0.

- λ is interpreted at the average increase in the value of the match (match specific capital).

- Divorce if value of divorce, V_d, is larger than θ_2:

$$\begin{aligned} \theta_2 &< E \max\{s, \theta\} = V_d \qquad (4) \\ \theta_1 + \lambda + \omega &< V_d \\ \omega &< V_d - \lambda - \theta_1 \end{aligned}$$

- $\Omega(\theta_1) = V_d - \lambda - \theta_1$. $\Omega(\theta_1)$ is the reservation match value for remaining married in the second period.

- If $\omega < \Omega(\theta_1)$, then the pair gets divorced. $\Omega(\theta_1)$ is decreasing in λ.

- The divorce may result in a remarriage or not.

- Given that she marries in the first period with a match value, θ_1, her expected utility in the second period is:

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$$V_2(\theta_1, \lambda) = E(\max\{\theta_2, V_d\}|\theta_1) \quad (5)$$
$$= \widehat{G}(\Omega(\theta_1))(\theta_1 + \lambda + \Gamma(\omega, \theta_1)) + G(\Omega(\theta_1))V_d$$
$$= \widehat{G}(\Omega(\theta_1))(\theta_1 + \lambda + \Gamma(\omega, \theta_1))$$
$$+ G(\Omega(\theta_1))(\theta_1 + \lambda + \Omega(\theta_1)) \quad (6)$$
$$= \theta_1 + \lambda + \widehat{G}(\Omega(\theta_1))\Gamma(\omega, \theta_1) + G(\Omega(\theta_1))\Omega(\theta_1)$$
$$> \theta_1 + \lambda$$

where $\widehat{G}(\Omega(\theta_1)) = 1 - G(\Omega(\theta_1))$ and $\Gamma(\omega, \theta_1) = E(\omega|\omega > \Omega(\theta_1))$.

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- If divorce allows a couple to get out of a bad marriage, it is enough to show that divorce is welfare improving.

- Her total utility from marriage in the first period is:

$$V_m(\theta_1, \lambda) = \theta_1 + V_2(\theta_1, \lambda) \quad (7)$$

- If she does not marry in the first period, she will get:

$$V_s = s + E\,\mathsf{max}(s, \theta) = s + V_d \tag{8}$$

- So in the first period she will choose:

$$\begin{aligned} V(\theta_1) &= \mathsf{max}\{V_s, V_m(\theta_1, \lambda)\} \\ &= \mathsf{max}\{s + V_d, \theta_1 + V_2(\theta_1, \lambda)\} \end{aligned} \tag{9}$$

- Since $V_2(\theta_1, \lambda) > V_d$, she may choose to marry even when $\theta_1 < s$.

- First, due to the accumulation of marriage specific capital, she expects the match value in the second period to be higher than from meeting someone new.

- Second, even if $\lambda = 0$, when there is no accumulation of marriage specific capital, marrying allows her to see how the match will evolve. If the match evolves poorly, she can divorce and try to match with someone else.

4 Under investment in marriage specific capital

- Assume that a man, h, and a woman, w, with match value θ_1 decides to marry.

- Let $\lambda = e_h + e_w$, where e_i is the investment of individual i in the first period of marriage.

- Let the cost of investment to individual i be e_i^2.

- Each individual will choose effort independently.

- So we want to find the Nash equilibrium (NE) in effort contribution.

- Let $\Omega(\theta_1, e_h, e_w) = V_d - \theta_1 - e_h - e_w$.

- Using (??), the efficient solution, which maximizes total marital output minus costs, solves:

$$\max_{e_h,e_w}\{2V_2(\theta_1, e_h + e_w) - e_h^2 - e_w^2\} \quad (10)$$

$$= \max_{e_h,e_w}\{2[\theta_1 + e_h + e_w \quad (11)$$

$$+\widehat{G}(\Omega(\theta_1, e_h, e_w))E(\omega|\omega \geq \Omega(\theta_1, e_h, e_w)) \quad (12)$$

$$+ G(\Omega(\theta_1, e_h, e_w))\Omega(\theta_1, e_h, e_w)] - e_h^2 - e_w^2\}$$

- Let e_i^* be the efficient choice of effort for individual i.

- The first order condition for e_i is:*

$$2\frac{\partial V_2(\theta_1, e_i^* + e_{\neq i}^*)}{\partial e_i^*} = 2e_i^* \tag{13}$$

$$1 - G(V_d - \theta_1 - e_i^* - e_{\neq i}^*) = e_i^*$$

- The left hand hand side of (13) is the marginal benefit of effort.

- The right hand side is the marginal cost of effort.

- Look for a symmetric equilibrium in which $e_i^* = e_{\neq i}^* = e^*$.

*Given a random variable ω with cumulative distribution $G(\omega)$, if

$$Y(x) = G(h(x))h(x) + (1 - G(h(x)))E(\omega|\omega \geq h(x))$$

then

$$\frac{\partial Y}{\partial x} = G(h(x))h'(x)$$

- (13) becomes:

$$2\frac{\partial V_2(\theta_1, 2e_i^*)}{\partial e_i^*} = 2e_i^* \tag{14}$$

$$1 - G(V_d - \theta_1 - 2e_i^*) = e_i^* \tag{15}$$

- We can solve for e^*, the efficient level of effort.

- In the Nash equilibrium, each agent takes the other's effort level as given, $e_{\neq i}^n$, and solve

$$\max_{e_i}\{V_2(\theta_1, e_i + e_{\neq i}^n) - e_i^2\} \tag{16}$$

- The first order condition for effort in this case is:

$$\frac{\partial V_2(\theta_1, e_i^n + e_{\neq i}^n)}{\partial e_i^n} = 2e_i^n \tag{17}$$

- Look for a symmetric equilibrium in which $e_i^n = e_{\neq i}^n = e^n$.

- (17) becomes:

$$\frac{\partial V_2(\theta_1, 2e^n)}{\partial e^n} = 2e^n \quad (18)$$

$$\frac{1 - G(V_d - \theta_1 - 2e_i^n)}{2} = e_i^n \quad (19)$$

- Comparing (14) with (18), the marginal costs of effort are the same in the NE and in the efficient case.

- But the marginal benefits of effort are twice as large in the efficient case than in the NE.

- Thus individuals will invest less in the NE than in the efficient case.

- Can we change divorce cost to get to a more efficient solution?