1 Uniform distribution

Let x, the match value, be a random variable with a uniform distribution. x ranges from 0 to 1.

The density of x is $f(x) = 1$.

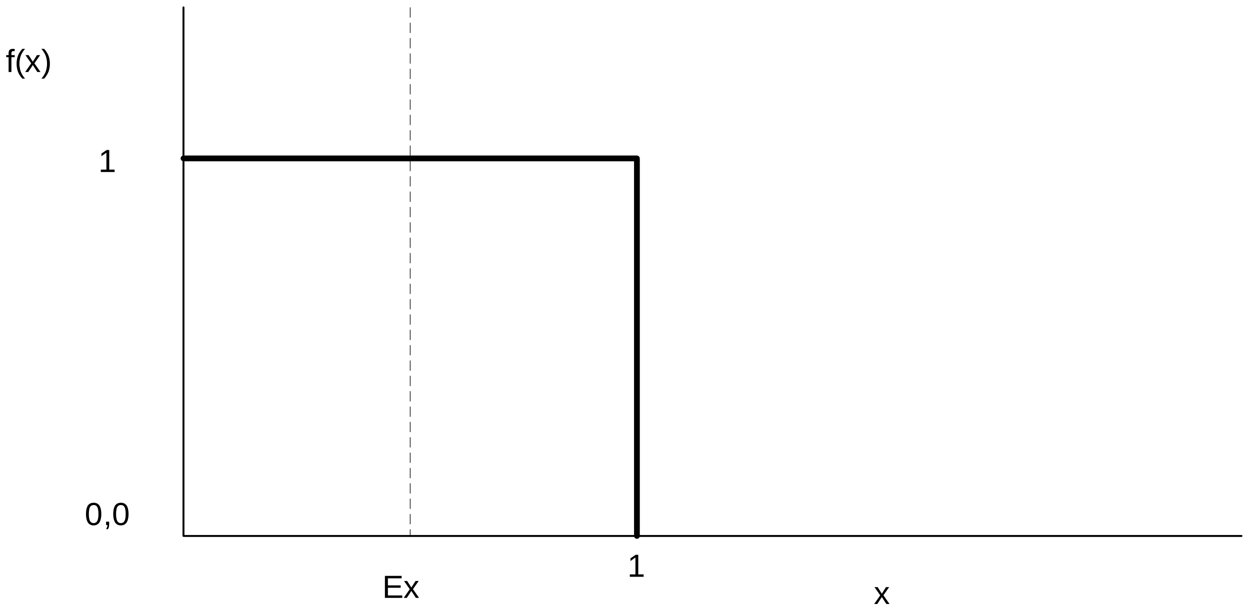
What is the probability that $x \leq X$?

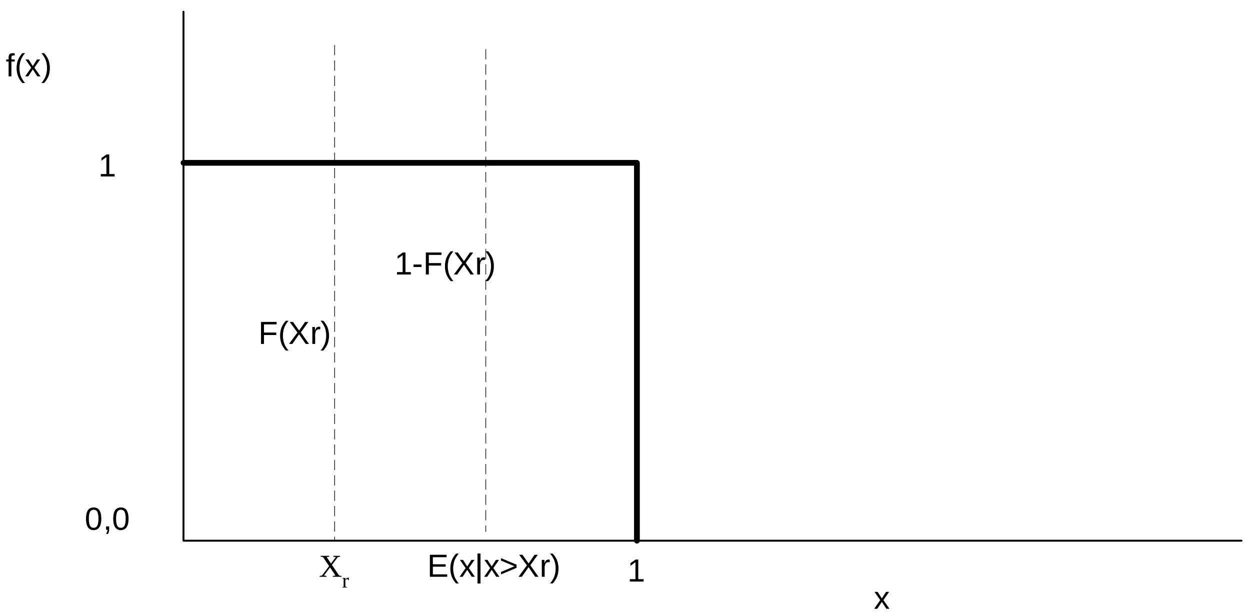
This probability, $F(X)$, is also known as the cumulative distribution of x. For the uniform distribution,

$$F(X) = \int_0^X f(x)dx = \int_0^X dx = X \tag{1}$$

The expected value of x is

$$E(x) = \int_0^1 xf(x)dx = \int_0^1 xdx = \frac{1}{2} \tag{2}$$


f(x)
1
0,0
Ex
1
x


f(x)
1
1-F(Xr)
F(Xr)
0,0
Xr
E(x|x>Xr)
1
x

Let $X_r > 0$ be the reservation match value. Then

$$E(x|x \geq X_r) = \frac{\int_{X_r}^{1} x f(x) dx}{\int_{X_r}^{1} f(x) dx} = \frac{1 + X_r}{2} > \frac{1}{2} \quad (3)$$

$$\begin{aligned} E \max(x, X_r) &= F(X_r) X_r + (1 - F(X_r)) E(x|x \geq X_r) \\ &= X_r^2 + (1 - X_r) \frac{1 + X_r}{2} = \frac{1 + X_r^2}{2} \quad (4) \end{aligned}$$