Marriage and Cohabitation in Canada

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Abstract

Marriage matching functions are used to study how marital patterns change when population supplies change. A behavioral marriage matching function with spillover effects is used to rationalize marriage and cohabitation behavior in contemporary Canada. The model non-parametrically identifies the systematic gains to marriage and cohabitation for a couple relative to remaining single. These gains are the primitives of the model and hence invariant to changes in the number of single men and women in the marriage market. We use the estimated model to quantify the impacts of gender differences in mortality rates and the baby boom on observed behavior. The higher mortality rate of men makes men more scarce relative to women. We show that this modestly reduced (increased) the welfare of women (men) in the marriage market. On the other hand, the baby boom increased the net gains to entering the marriage market for older men and lowered the net gains for middle aged women.

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Demographers have long been interested in studying marital matching or who marries whom. Most recent empirical work use log linear models to study marital matching by race, education, nationality, religion and other attributes (E.g. Kalmijn 1991a, 1991b, 1993; Mare 1991; Qian 1997; Rosenfeld 2001). The main advantage of a saturated log linear model is that it is non-parametric, and it can fit any observed marriage pattern. Log linear models can be used to test different hypotheses about matching patterns such as the presence of endogamy for a particular type of individuals. As a marriage matching model, log linear models have two important disadvantages. First, individuals who choose not to marry are ignored in the analysis. Second, because the model has no explicit dependence on population characteristics, the estimated models cannot be used for making forecasts of how marriages would respond to changes in the numbers of different types of individuals in the marriage market. ¹ Because of this, standardization techniques cannot be applied to log linear models to make forecasts. Rather the primitive in a log linear model of marital patterns is the number of marriages, which is taken as exogenous.

Schoen’s 1981 harmonic mean marriage matching model generalizes the log linear model and explicitly links marital matching to the supplies (or population) of different types of individuals in the marriage market.² His model accommodates unmarried individuals. It is also non-parametric and can fit any observed marriage matching pattern. An estimated Schoen’s model can be used to make forecasts of changes in marital matching patterns due to changes in the population or supply of different types of individuals. However, the Schoen’s model has a serious disadvantage, it assumes that the number of marriages between two types of individuals are unaffected by changes in the number of other types of individuals. For example, the number of nineteen year old men who choose to marry eighteen year old women is assumed to be unaffected by changes in the number of twenty year old men. That is, Schoen’s model ignores spillover or substitution effects across types of individuals. Demographers have long recognized the importance of spillover (or substitution) effects in marriage matching functions (McFarland 1972; Pollard 1997). The analytic problem has been to specify marriage matching functions which include substitution effects and yet remain statistically identified (e.g. Pollak 1990a).

The objective of this paper is to review and extend the Choo Siow (2005) marriage matching model (hereafter CS), a marriage matching model with spillover effects. Like the log linear and Schoen’s model, the CS model is non-parametric and can fit any marriage

¹An exception is when individuals are indifferent to whom they marry. That is, there are only main effects in the log linear model.

²Preston and Qian (1993) provide a nice motivation for Schoen’s model which we will discuss below.
distribution. The model provides a behavioral interpretation of marital matching patterns. This behavioral interpretation provides a formal rationalization of standard interpretations of marriage matching patterns in the literature.

We extend the CS model to include cohabitation and use it to study marriage and cohabitation in Canada. Marriage and cohabitation are common adult living arrangements in Canada. Figure 1 shows the age distributions of adult Canadians in 1996 and their living arrangements. Unlike marriage, cohabitation occurs primarily among adults below the age of thirty. Older Canadians are more likely to be married or single. Since cohabitation occurs primarily among young adults, its share of adult living arrangements depends on the age distribution of the population. As shown in Figure 1, a prominent feature of the age distributions is the temporal coincidence between the arrival of the baby boom generation into adulthood and the rise of cohabitation. This raises the question: How has the baby boom generation affected cohabitation in Canada?

Although both marriage and cohabitation are heterosexual unions, the age pattern of living arrangements of men and women are substantially different. Figure 2 shows, by age, the logs of the ratio of number of men (women) relative to the number of men (women) who are single. The share of young women in coupled living arrangements exceeds that of young men. The share of older women in similar living arrangements is less than that of older men. How much of this age specific gender differences in living arrangements are due to the higher mortality rate of men, and how much must be explained by other marriage market factors? We extend the CS model to address who gains and looses in the marriage and cohabitation market from differential gender mortality and the baby boom.

The rest of the paper is organized as follows. In the next section we review marriage matching functions. This is followed by a description of the CS marriage matching function. We highlight the differences between the CS matching function and the Schoen’s matching function using a numerical example. The extension of the CS model to allow for cohabitation

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3We ignore same sex unions in this paper because our empirical analysis is based on the 1996 Census which has no data on these unions.

4Cohabitation is defined as the answer “yes” to the following question in the 1996 Census: “Is this person living with a common-law partner? (Common-law refers to two people who live together as husband and wife but who are not legally married to each other.)”. The appendix provides detailed data construction and sources.

5There are many reasons for the rise of cohabitation. We do not attribute its increase to the arrival of the baby boom. But rather ask how has changes in the population due to the arrival of the baby boom affected the attractiveness of cohabitation within the CS model.

6This statistic is a monotone tranformation of the more familiar “marriage & cohabitation” rate. We will show why this is a more useful statistic in the paper.
follows. In the empirical application we look at how gender differences in mortality rates and the baby boom affects cohabitation and marriage in Canada in 1996. This is followed by a conclusion.

**Marriage matching models**

The marriage market is a bilateral matching market with heterogenous participants.\(^7\) Let there be \(I\) different types of men and \(J\) different types of women. An individual’s type is characterized by his or her attributes such as age, ethnicity, religion, education and other attributes that the analyst wants to study.

Let \(M\) be the vector of available men with types \(i = 1, \ldots, I\) at a given point in time, where \(m_i\), the \(i^{th}\) element of \(M\), is the number of available type \(i\) men. Similarly, \(F\) denotes the vector of available women with types, \(j = 1, \ldots, J\) with the \(j^{th}\) element \(f_j\) referring to the number of available type \(j\) women. Let \(\Pi\) be a matrix of parameters. A marriage matching function, \(\mu(M, F; \Pi)\), returns a \(I \times J\) matrix, where the \((i, j)\) element, \(\mu_{ij}\), is the number of marriages between type \(i\) men and type \(j\) women at that time. The matrix \(\mu\) is also referred to as the marriage distribution. We denote the number of unmarried men of type \(i\) and unmarried women of type \(j\) as \(\mu_{i0}\) and \(\mu_{0j}\) respectively. \(N\) denotes the total number of marriages, \(\sum_{j=1}^{J} \sum_{i=1}^{I} \mu_{ij}\). The marriage matching function, \(\mu(M, F; \Pi)\) must satisfy:

\[
\mu_{0j} + \sum_{i=1}^{I} \mu_{ij} = f_j \quad \forall \, j
\]

\[
\mu_{i0} + \sum_{j=1}^{J} \mu_{ij} = m_i \quad \forall \, i
\]

\[
\mu_{0j}, \, \mu_{i0}, \, \mu_{ij} \geq 0 \quad \forall \, i, j
\]

Equations (1), (2) and (3) are accounting constraints. (1) says that the total number of men who marry \(j\) type women and the number of unmarried \(j\) type women must be equal to the number of available \(j\) type women for all \(j\). Similarly (2) says that the total number of women who marry \(i\) type men and the number of unmarried \(i\) type men must be equal to the number of available \(i\) type men for all \(i\). (3) holds because the number of unmarries of any type and gender, and the number of marriages between type \(i\) men and type \(j\) women must be non-negative.

The log linear marriage matching function assumes that the number of marriages is independent of the number of unmarrieds, \(M\) and \(F\), and takes the total number of marriages,

\(^7\)This discussion borrows from Pollak 1990a and 1990b.
\( T \), as an exogenous parameter of the function. The number of marriages between type \( i \) men and type \( j \) women, \( \mu_{ij}(M, F; \Pi) \), takes the form,

\[
\mu_{ij}(M, F; \Pi) = \mu_{ij}(T; \Pi) = T \exp(\pi_{ij}).
\]

Given data on marriages, we take natural logs to arrive at the familiar form

\[
\ln\left(\frac{\mu_{ij}}{T}\right) = \pi_{ij}. \tag{4}
\]

The log linear marriage matching function of (4) expresses the proportion of \((i, j)\) marriages, \( \frac{\mu_{ij}}{T} \), as a function of the parameter \( \pi_{ij} \). Being a proportion, the following restriction applies: \( \sum_{i=1}^{I} \sum_{j=1}^{J} \exp(\pi_{ij}) = 1 \). The saturated log linear marriage matching function can fit any observed marriage distribution \( \mu \). A common parameterization of (4) is to decompose \( \pi_{ij} \) as,

\[
\pi_{ij} = \beta + \beta_i + \beta_j + \beta_{ij} \tag{5}
\]

with the restrictions:

\[
\sum_{i=1}^{I} \beta_i = 0, \quad \sum_{j=1}^{J} \beta_{ij} = 0 \quad \forall \ i
\]

\[
\sum_{j=1}^{J} \beta_j = 0, \quad \sum_{i=1}^{I} \beta_{ij} = 0 \quad \forall \ j
\]

The parameters \( \beta_i \) and \( \beta_j \) are commonly known as the main effects, and \( \beta_{ij} \) as the secondary effects. (5) is convenient because different hypotheses about marital matching patterns can usually be formulated as further restrictions on the secondary effects, \( \beta_{ij} \).

While the log linear model is convenient for hypotheses testing, in general it cannot be used for forecasting. Its formulation inherently assumes that marriages does not depend on the supplies of available men and women, \( M \) and \( F \), and that the total number of marriages, \( T \), is exogenously predetermined. Without a model of how \( T \) will respond to changes in \( M \) and \( F \), we cannot predict how the marriage distribution \( \mu \) will respond to changes in \( M \) and \( F \).\(^8\)

As alternatives, demographers usually work with matching functions with a zero spillover matching rule (Pollak 1990b):

\[
\mu_{ij}(M, F; \Pi) = \mu_{ij}(m_i, f_j; \bar{\pi}_{ij})
\]

\(^8\)Log linear models were originally constructed to interpret contingency tables where each dimension of the table refers to a different attribute of an observation. \( T \) refers to the number of observations which can be regarded as exogenous. In its original use, there is no meaning to increasing the number of observations along one attribute holding the number of observations along the other attributes constant.
That is, the number of \( i, j \) matches only depends on the number of available type \( i \) men and type \( j \) women, \( m_i \) and \( f_j \). Schoen’s 1981 harmonic mean mating rule, the current workhorse in demography takes the form,

\[
\mu_{ij}(m_i, f_j; \tilde{\pi}_{ij}) = \frac{m_i f_j}{m_i + f_j} \delta_{ij}.
\]

(7)

Taking logs of both sides and letting \( \delta_{ij} = \exp(\tilde{\pi}_{ij}) \), we arrive at the familiar form,

\[
\ln \mu_{ij} = \ln \left[ \frac{m_i f_j}{m_i + f_j} \right] + \tilde{\pi}_{ij},
\]

(8)

where \( \sum_j \exp(\pi_{ij}) \leq 1 \), and \( \sum_i \exp(\pi_{ij}) \leq 1 \) is a zero spillover matching rule. This matching function will satisfy the accounting constraints, (1), (2) and (3). The marriage matching function is non-parametric and it fits any marriage distribution \( \mu \). Like the log linear model, we can also parameterize \( \pi_{ij} \) as in equations (5) and (6). Thus, we can test different hypotheses about marital matching patterns as further restrictions on \( \beta_{ij} \) as in the log linear model. However, the Schoen’s model has one major advantage over the log linear model. Given estimates of \( \pi_{ij} \), equation (8) can be used to make predictions about marital behavior given new population vectors \( M' \) and \( F' \).

Qian and Preston (1993) provides a nice motivation for Schoen’s model as follows. Let \( m_i \) males and \( f_j \) females enter the \( i, j \) room. Each individual will randomly meet with another individual in the room. The probability that a male will meet a female is \( f_j / (m_i + f_j) \). The expected total number of meetings between males and females in that room is \( m_i f_j / (m_i + f_j) \). If \( \delta_{ij} \) is the rate or proportion of these meetings or encounters that will lead to marriage, we obtain (7), Schoen’s marriage matching function.

While zero spillover marriage matching functions are easy to estimate and use, the zero spillover assumption is restrictive. Holding the parameters of the marriage matching function, \( \delta_{ij} \)’s, constant, changes in \( m_{i'} \) and \( f_{j'} \) where \( i' \neq i \) or \( j' \neq j \) do not affect \( \mu_{ij} \). For example, the number of nineteen year old males married to nineteen year old females is unaffected by variation in the supplies of twenty year old males or females. Following the discussion of the CS model in the next section, we provide a numerical example comparing these two matching functions and the implied spillover effects.

Demographers have of course recognized the importance of spillover (or substitution) effects in marriage matching functions. The problem has been to specify marriage matching functions which include substitution effects and yet remain statistically identified. Pollard and Höhn 1993/94 provide the most sophisticated matching function of this kind:

\[
\mu_{ij}(M, F) = \frac{m_i f_j a_i b_j}{\frac{1}{2} (\sum_k m_k a_k h_{kj} + f_k b_k h_{ik})},
\]
where $a_i$, $b_j$, and $h_{ij}$ are weight functions that are specified by the analyst. When $i$ and $j$ refer to the ages of the participants, the types of individuals are ordered by age. Using this natural ordering, Pollard and Höhn suggested some plausible weight functions. But if types are also defined by ethnicity, religion and other attributes that are not naturally ordered, then it is difficult to a priori specify the weight functions. But without a priori restrictions on the weights, the model is not identified.

**The CS marriage matching function**

The CS marriage matching function has spillover effects. Following the main advantage of the log linear and Schoen’s model, the CS model is also non-parametric and can fit any observed marriage distribution. Although $\mu_{ij}$ depends on the number of available men and women competing for a spouse, $M$ and $F$, this dependence does not introduce any additional parameters above what is already present in the two preceding models. Hence, the CS model remains identified.

The CS model is:

$$
\ln \left[ \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \right] = \ln \left[ \frac{\mu_{ij}}{\sqrt{(m_i - \sum_k \mu_{ik})(f_j - \sum_l \mu_{lj})}} \right] = \pi_{ij} \quad (9)
$$

Given observed quantities, $\mu_{ij}$, $\mu_{i0}$ and $\mu_{0j}$, the parameter $\pi_{ij}$ can be estimated. Substitution (spillover) effects are present in the matching function. That is, $\mu_{ij}$ will change if $m_{i'}$ or $f_{j'}$ change even if $i \neq i'$ or $j \neq j'$. Put another way, $\frac{\partial \mu_{ij}}{\partial m_k} \neq 0$ and $\frac{\partial \mu_{ij}}{\partial f_l} \neq 0$ for all $k,l$. We can also parameterize $\pi_{ij}$ as (5) and (6). The analyst does not have to a priori specify weighting functions for substitution effects that the analyst may have little prior information about. Since there are no additional parameters to be estimated, although present, the substitution patterns embodied in the CS model are restrictive. Whether the substitution patterns are empirically reasonable remains to be determined.

The CS model provides a behavioral interpretation of $\pi_{ij}$. It is an empirical implementation of Becker’s transferable utility model of the marriage market. CS (2005) provides a formal derivation of (9). We focus here on providing intuition for the result.

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9Dagsvik (2000) derived a closely related marriage matching function using the same random utility functions as we do in a non-transferable utilities setting. See CS (2005) for a more detailed comparison of the two frameworks. The main empirical difference is that his marriage matching function has increasing returns in population supplies. With data from a single cross section as we have in this paper, the two models cannot be distinguished.

10CS provides a matrix representation of these substitution effects.
In transferable utility models of the marriage market, a marriage between two individuals is assumed to generate marital output which depends on the identities of the two individuals. If the two individuals choose to marry each other, the marital output is divided between them in some way. All types of marriage matches are potentially feasible. The above conception raises two questions. First, how is the marital output within a marriage divided? Second, with many different types of men and women, who marries whom?

Modelling the marriage market as a large competitive market for all types of marital matches provides an answer to both questions. Following standard competitive market models, we will derive demand and supply curves for spouses for every potential type of marital match, and then find the equilibrium prices which will clear the marriage market. The equilibrium (market clearing) prices divide the marital output between spouses. One concern about marriage market models is what corresponds to market clearing prices in the real world. We interpret these market clearing prices as each spouse’s share of responsibilities within a marriage.\(^{11}\)

Providing more detail, if a male of type \(i\) wants to marry a female of type \(j\), he has to be willing to transfer \(\tau_{ij}\) of marital output to her. If a female of type \(j\) wants to marry a male of type \(i\), she has to be willing to accept a transfer \(\tau_{ij}\) of marital output from him. Since the transfer can be positive or negative, there is no substantive consequence from assuming that husbands pay the transfers and wives receive them. The marriage market clears when the equilibrium transfers, \(\tau_{ij}\), are such that, for all \((i, j)\) pair, all type \(i\) men who want to marry type \(j\) women do so and vice versa. The matches that occur maximizes the sum of total marital output in the society.\(^{12}\) In equilibrium, men and women may remain unmarried.

Since the same type of men may marry different types of women and vice versa, we need a model where different individuals of the same type will make different marital decisions. CS (2005) assumes that a female individual \(k\) of type \(j\) who marries a type \(i\) male will receive the sum of three payoffs: (1) The spousal contribution to marital output by a type \(j\) female married to a type \(i\) male, \(\gamma_{ij}\). (2) The transfer that she will receive from her husband, \(\tau_{ij}\). And (3), an idiosyncratic payoff that is particular to her being married to a type \(i\) male, \(\varepsilon_{kij}\). The first two payoffs are non-stochastic and common to all type \(j\) women who marry type \(i\) men. The third payoff is purely idiosyncratic and accrues only to individual \(k\) if she marries a male of type \(i\). \(\varepsilon_{kij}\) is hence a random variable. Different realizations of these idiosyncratic

\(^{11}\)There are further issues about whether spouses can renege on their promised responsibilities after marriage and what happens when unexpected contingencies arise after marriage. We work within a static framework and therefore cannot deal with these dynamic issues.

\(^{12}\)They are also the matches that maximize the welfare of a utilitarian social planner.
payoffs across different individuals of the same type generate differences in observed marital behavior by different individuals of the same type. More formally, the random utility that female $k$ of type $j$ gets from a match with a type $i$ man is given by,

$$V_{kij} = \gamma_{ij} + \tau_{ij} + \varepsilon_{kij}. \quad (10)$$

It is important to note that she gets the same idiosyncratic payoff $\varepsilon_{kij}$ as long as she marries a type $i$ male. Put another way, from her perspective, all type $i$ males are perfect substitutes. This perfect substitutability assumption allows us to condition the transfer that she will receive in an $i,j$ marriage to only depend on $i$ and $j$ and not the specific identities of the two spouses.

If female $k$ of type $j$ decides not to marry, she will get her mean or common contribution to unmarried output $\gamma_{0j}$ and an idiosyncratic payoff $\varepsilon_{0jk}$ associated with remaining unmarried. She will not receive a transfer. Formally, the random utility that she receives from remaining single is,

$$V_{k0j} = \gamma_{0j} + \varepsilon_{k0j}.$$

Every type $j$ woman will decide whether to marry at all and which type of man to marry. She will choose the action that gives her the highest total payoff, accounting for the common spousal contribution, the transfer if any, and the idiosyncratic payoff. Idiosyncratic payoffs across different choices and different women are assumed to be independent and identically distributed. Different type $j$ women who choose to marry type $i$ men will get the same common spousal contributions $\gamma_{ij}$ and the same transfers $\tau_{ij}$. But they will each get different idiosyncratic payoffs depending on the $\varepsilon_{kij}$ drawn for that choice. This makes their total payoffs from marrying type $i$ men different. It is therefore this different idiosyncratic payoffs that will make two women of the same type $j$ make different choices. Notice also that different types of women who marry type $i$ men will get different common payoffs and transfers. Thus, there will be systematic differences in marital choices by different types of women.

We assume that all the independent and identically distributed idiosyncratic payoffs are drawn from a type 1 extreme value distribution.\(^{13}\) McFadden(1973) showed that,

$$\ln \left[ \frac{\mu^s_{ij}}{\mu_{0j}} \right] = \gamma_{ij} - \gamma_{0j} + \tau_{ij} \quad (11)$$

where $\mu^s_{ij}$ are the number of type $j$ women who want to marry type $i$ men and and $\mu_{0j}$ the number who remain single. Equation (11) is a quasi supply curve of type $j$ wives to

\(^{13}\)Formally, the optimization problem that individual $k$ faces is represented by $V_{kj} = \max\{V_{k0j}, V_{k1j}, \ldots, V_{kIj}\}$. See CS (2005) for details.
type $i$ men. \(? \mu_{ij} \) is the log odds of type $j$ women marrying type $i$ men relative to those remaining unmarried. If the common or average payoffs to type $j$ women marrying type $i$ men (relative to remaining single) is large, this would be reflected by more $(i,j)$ matches relative to the number of $j$ type women who chose to remain single as captured by the log odds ratio. (11) says that as the female marital contribution increases relative to the return to remaining single, and as the transfers females receive increase, the proportion of those marriages relative to the proportion remaining single also increases. Remember that all type $j$ women receive an average payoff $\gamma_{ij} + \tau_{ij}$ if they marry type $i$ men. So as their common payoff to type $i$ marriages rises relative to their common payoff from remaining single, then the log odds of type $j$ women marrying type $i$ men relative to remaining unmarried should increase. The functional form of the quasi supply curve in (11) owes its simplicity to the additively separable structure of the marriage payoffs as well as the specific distribution of the idiosyncratic payoffs. While other more general assumptions may provide similar qualitative results, they are not as analytically tractable.

CS models the behavior of men analogously. A particular man $g$ of type $i$ who marries a type $j$ female will receive the sum of three payoffs: (1) The spousal marital output contribution by a type $i$ male married to a type $j$ female, $\alpha_{ij}$. (2) The transfer that he will pay his wife, $\tau_{ij}$. And (3), an idiosyncratic payoff that is particular to him being married to a type $j$ female, $\epsilon_{gij}$. The first two payoffs are common to all type $i$ men who marry type $j$ women. The third payoff is purely idiosyncratic and accrues only individual $g$ if he marries a female of type $j$. Formally, his random utility function from marrying a type $j$ female is

$$W_{gij} = \alpha_{ij} - \tau_{ij} + \epsilon_{gij}. \quad (12)$$

If male $g$ of type $i$ decides not to marry, he will get his contribution to unmarried output $\alpha_{i0}$. He does not pay any transfer but gets an idiosyncratic payoff $\epsilon_{i0}$ associated with remaining unmarried. The random utility from remaining unmarried is

$$W_{g0} = \alpha_{i0} + \epsilon_{g0}.$$

Every type $i$ man will decide which type of woman to marry or whether to marry at all. He will choose the action that gives him the highest total payoff, accounting for the sum of spousal contributions, the transfers he pays if any and the idiosyncratic payoffs. Idiosyncratic payoffs are assumed to be independent and identically distributed across different choices and individuals. We maintain the same distributional assumption on $\epsilon_{gij}$ as in the specification of the female decision problem. Taking the log ratio of the proportion of $(i,j)$ matches relative to the proportion of $i$ men who remain unmarried, we get a quasi demand curve for wives.
of type $j$ by type $i$ men,

$$\ln \left[ \frac{\mu_{ij}^d}{\mu_{i0}} \right] = \alpha_{ij} - \alpha_{i0} - \tau_{ij}. \quad (13)$$

$\mu_{ij}^d$ is the number of type $i$ men who wants to marry type $j$ women and $\mu_{i0}$, the number who remain unmarried. The term on the left hand side, $\ln \left[ \frac{\mu_{ij}^d}{\mu_{i0}} \right]$, is the log odds of type $i$ men marrying type $j$ women relative to remaining unmarried. Analogous to the quasi-supply equation for females, if the average common marital payoff to an $i$ type man marrying a type $j$ women relative to his common payoffs from remaining single is large, this would be reflected in more $(i, j)$ marriages relative to the number of $i$ type men who chose to remain single as captured by the log odds ratio. (13) also says that the log odds ratio of $(i, j)$ marriages relative to the number of $i$ type men remaining single also increases as $(\alpha_{ij} - \alpha_{i0})$, the common marital output contribution relative to remaining single increases, and as $\tau_{ij}$ the transfers men pay fall.

Any empirical distribution of marriages $\mu$ given a population of single men and women, $M$ and $F$, can be thought of as a realization of a marriage market equilibrium. In this equilibrium, the marriage market clears when, given transfers $\tau_{ij}$ for all $I \times J$ markets, the demand for spouses is equal to the supply of spouses in each of these markets. That is,

$$\mu_{ij}^d = \mu_{ij}^s = \mu_{ij} \quad \forall i, j. \quad (14)$$

We can substitute the market clearing condition of (14) into the quasi supply and demand equations, (11) and (13) to get,

$$\ln \left[ \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \right] = \gamma_{ij} + \alpha_{ij} - (\alpha_{i0} + \gamma_{0j}) \quad \pi_{ij} \quad (15)$$

which is the CS marriage matching function. $\pi_{ij}$ represents the total gains to marriage and is estimated by the statistic $\ln \left[ \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \right]$. It measures the expected per spousal marital output from an $i, j$ marriage relative to both parties remaining unmarried. (15) is intuitive. When $\pi_{ij}$, the total gains to marriage is high, the number of $i, j$ marriages, $\mu_{ij}$, is high relative to the geometric average of the number of unmarrieds, $\sqrt{\mu_{i0}\mu_{0j}}$. This argument again relies on the fact the ratio of observed $(i, j)$ marriages relative to the number of each type who chose to remain single provides information about the average total gain from such matches relative to remaining single.

If there are many unmarried $i, j$ individuals and few $i, j$ marriages, the geometric average of the unmarrieds $\sqrt{\mu_{i0}\mu_{0j}}$ will exceed $\mu_{ij}$. In this case, $\pi_{ij}$ will be negative. Such an outcome is normal. It says that the expected per spousal marital output from an $i, j$ type marriage is
less than the average of what the parties may obtain by remaining unmarried. For example, if \( i \) refers to 20 year old males and \( j \) refers to 40 year old females, \( \pi_{20,40} \) is negative. Why would there be marriages for matches in which \( \pi_{ij} \) are negative? Even though the expected marital output relative to remaining unmarried is negative (i.e. for most \( i,j \) couples), it is not negative for all \( i,j \) matches. Some \( i,j \) individuals will derive large positive idiosyncratic payoffs from those matches and therefore choosing an \( i,j \) marriage maximizes their utilities.

In marriage market equilibrium,

\[
\begin{align*}
n_{ij} &= \ln \mu_{ij} - \ln \mu_{i0} = \alpha_{ij} - \alpha_{i0} - \tau_{ij}, \quad (16) \\
N_{ij} &= \ln \mu_{ij} - \ln \mu_{0j} = \gamma_{ij} - \gamma_{0j} + \tau_{ij}, \quad (17)
\end{align*}
\]

\( n_{ij} \) and \( N_{ij} \) measure the expected net gain to an \( i \) type man in a \( i,j \) match relative to remaining single and the expected net gain to a \( j \) type woman in a \( i,j \) match relative to remaining single respectively. Again, the interpretation is intuitive. If there are more type \( i \) men in an \( i,j \) match than in a \( i,j' \) match, we expect the net gains to a \( i \) type man in a \( i,j \) match is larger than his expected net gain in a \( i,j' \) match.

CS (2005) also shows that the marriage rates

\[
\begin{align*}
\sum_k \mu_{ik} &= \ln \left( \frac{m_i}{\mu_{i0}} \right) \approx \ln(\frac{m_i}{\mu_{i0}}), \quad (18) \\
\sum_k \mu_{kj} &= \ln \left( \frac{f_j}{\mu_{0j}} \right) \approx \ln(\frac{f_j}{\mu_{0j}}) \quad (19)
\end{align*}
\]

measure the expected benefit to a type \( i \) male and type \( j \) female respectively from being able to participate in the marriage market as opposed to being forced to remain single.\(^{14}\)

Of course, being able to participate does not mean that every participant will choose to marry. Rather, it means being able to choose between marriage or remaining single whereas non participation means having to remain single. Again, the measured quantities have intuitive appeal. If \( \mu_{i0} \) is large, approximately equal to \( m_i \), then the expected benefit from participating in the marriage market, \( \ln \left( \frac{m_i}{\mu_{i0}} \right) \approx 0 \) which should be expected. If most males of type \( i \) choose to remain unmarried, then the expected benefit to type \( i \) male of being able to participate in the marriage market is small. Similarly, if \( \ln \left( \frac{f_j}{\mu_{0j}} \right) \) is large, then it is very costly for a type \( j \) female to be excluded from the marriage market. The higher the marriage rate, the larger the gains to marriage for type \( j \) females relative to not being able to enter the

---

\(^{14}\)This approximation is valid since

\[
\ln \left( \frac{m_i}{\mu_{i0}} \right) = -\ln \left( 1 - \frac{\sum_k \mu_{ik}}{m_i} \right) \approx \frac{\sum_k \mu_{ik}}{m_i}.
\]
marriage market. Social scientists often assume that the marriage rate is a good proxy for the gains to marriage. For example, the Wilson’s hypothesis explains the low marriage rate of black females compared with white females in urban environments as due to the expected low gains to marriage for black females in these environments (see Wilson(87)). The CS framework provides a formal justification for these interpretations of the marriage rate.

A feature of the model is that we cannot recover from the data is $\tau_{ij}$, the equilibrium transfers which clear the marriage market. Consequently, this also means that we cannot separately recover the male and female marital contributions $(\alpha_{ij} - \alpha_{i0})$ and $(\gamma_{ij} - \gamma_{0j})$ respectively.

For Schoen’s model (8), Preston and Qian interprets $\pi_{ij}$ as the force of attraction, which is similar to our total gains to marriage. Schoen predicts that $\mu_{ij}$ is proportional to the harmonic mean of the supplies of $i$ and $j$ whereas CS predicts that it is proportional to the geometric mean of the unmarrieds. Other than functional form differences, there is also a conceptual difference. As Preston and Qian’s motivation show, Schoen’s formula can be derived from a model where individuals are first randomly assigned to a room where each room has one type of male and one type of female. In an $i, j$ room, individuals can marry each other or not at all. The CS model is derived from the assumption that $i, j$ individuals can marry each other or any other type, or not marry. The geometric average of the unmarrieds summarizes the opportunity cost of $i, j$ marrying each other.

As discussed earlier, Schoen’s model has no spillover effect while the CS model allows for spillover effects. What follows is a simple numerical example illustrating this difference. Consider a marriage market of two types men and women, young and old. The $i$ and $j$ index for males and females respectively, equals $d$ when the person is old and $y$ when the person is young. Hence, $m_y$ and $m_d$ denote the numbers of single young and old men respectively. Similarly $f_y$ and $f_d$ denote the numbers of young and old women respectively. Suppose the number of single men and women, and marriages in this market are shown in Table 1. There are 41 single men (of which are 26 are young and 15 are old) and 47 single women (30 young and 17 old women). The interactions of these individuals generated 16 marriages between young men and young women ($\mu_{yy}$), 4 marriages between young men and old women ($\mu_{yd}$), and so on.

Insert Table 1 here.

Given these observations of marriages and the number of single men and women, we can compute the parameters of the Schoen and CS matching functions, $\exp(\tilde{\pi}_{ij})$ and $\exp(\pi_{ij})$ respectively as defined by equations (7) and (15). These estimated gains to marriage or
'forces of attraction' rationalize this distribution of marriages.\textsuperscript{15} In this numerical example, men are scarce relative to women.

Consider a change in the population vectors where the number of available single young men increase from the initial 26 to 36. Using the estimated parameters, both marriage matching functions allow us to make predictions of what the marriage distribution would be like given this change in the number of available young men. The linear structure of the Schoen matching function makes predictions straightforward. Predictions using the CS marriage matching function requires numerically solving the system of equations (15).\textsuperscript{16} The predicted marriages as a result of this change is given by Table 2.

Insert Table 2 here.

As expected the Schoen matching function predicts that the increase in supplies of young males would lead to increase in marriages between young male-young females, and young males-old females. The larger increase is borne by marriages between young male-young females. This can be explain by the larger estimate of the gains to marriage from this match. The number of unmarried young males is also predicted to increase to 12.709. However, the lack of spillover effects means that the number of marriages involving old males remain unchanged.

Predicted marriages using the CS matching function in Table 2b tells a different story. The increase in young males increases the number of marriages involving young males, implying more young male-young female and young male-old female marriages. It also predicts lower old male-young female marriages and (slightly) higher old male-old female marriages. Females as a whole stand to gain from this change, the number of unmarried young and old females is expected to decrease. Old males however are disadvantaged by this change, since there are now more unmarried old males than before this change.

The females involved in these additional marriages between young males and young females come from two sources. First, the majority are young females before this change would have been unmarried. The second source are young females that would have married old males before this change occurred. In total, the number of unmarried young females

\begin{align*}
\hat{\exp(\pi)} &= \begin{pmatrix} 1.149 & 0.389 \\ 0.6 & 0.878 \end{pmatrix}, & \hat{\exp(\pi)} &= \begin{pmatrix} 2.309 & 0.667 \\ 1.5 & 2.021 \end{pmatrix}.
\end{align*}

\textsuperscript{15}For this numerical example, the estimated parameters for the Schoen and the CS marriage matching function respectively are

\textsuperscript{16}CS (2005) provides further details. Gauss and stata codes for solving the system are available from the authors on request. The stata codes will also be made available through the website $www.$ to be added.
falls. This implies a pattern of substitution. Some young women who would have previously
married old men now substitute to young men. This frees up some old men who now
substitute for matches with old females instead. This latter substitution is small, generating
an overall increase in the number of unmarried old males.

**Changes in sex ratios**

The parameters defining male and female average marital contribution, \(\alpha_{ij}, \alpha_{i0}, \gamma_{ij}\) and \(\gamma_{0j}\)
also define the preferences of individuals in the marriage market. They directly affect the
average gain or utility individuals receive from different matches. CS (2005) assume that
these parameters are primitives of the model and are thus unaffected by the number of single
males and females, \(m\) and \(f\). The proposed measure of the total average gains per spouse
from an \((i, j)\) marriage relative to both individuals remaining single, \(\pi_{ij}\), in equation (15) is
only a function of these preference parameters. Hence, it too is invariant to changes in the
number of males and females and the sex ratio.

This does not mean that changes in sex ratios and in the supplies of singles has no effect
on marriages or the gender specific net average gain, \(N_{ij}\) and \(n_{ij}\). On the contrary, relative
scarcity of single males and females as measured by sex ratios affect the equilibrium transfers,
\(\tau_{ij}\), that clear the marriage market. As shown in the random utility equations for different
match choices, (10) and (12), the equilibrium transfer directly affects the average gain or
utility from a match. Similarly, the equilibrium transfers also directly affect the male and
female net average gains defined in (16) and (17). In other words, changes in supplies of
singles and in the sex ratio affect how the marital output is shared between husbands and
wives within a marriage. These effects on the division of gains within marriage in turn affects
the relative attractiveness of the different matches available to a single individual. However,
these changes have no effect on the the average total gain from marriage, \(\pi_{ij}\).

As mentioned earlier, observing \(\pi_{ij}\) however, is not sufficient for us to identify the gender
specific common gains, \((\alpha_{ij} - \alpha_{i0})\) and \((\gamma_{ij} - \gamma_{0j})\), nor to identify the transfer, \(\tau_{ij}\). In other
words, knowing a measure of the total gains to a match is not sufficient to determine whether
men pay positive or negative transfers to women in equilibrium. As the preceeding numerical
example illustrates, this does not prevent us from making marriage predictions when supplies
of males and females change. Since the CS model identifies a measure of the net average
gain to males and females, this also allows us to identify changes in equilibrium transfers
resulting from changes in sex ratios and supplies of singles.

Consider again the numerical example from the preceeding section concerning a marriage
market with two types of males and females. The distribution of marriages and of unmarrieds in Table 1 allows us to calculate measures of net common gains by gender according to (16) and (17). These are tabulated in Table 3a and 3b.

Focusing on the net gains for males in Table 3a, we observe that $n_{yy} = \ln(\mu_{yy}/\mu_{y0})$, the net average gains to a young male from marrying a young female relative to remaining unmarried, is positive. This suggests that the average net gains that young males expect to receive from such a match are higher than the average net gains from remaining unmarried. Similarly, the negative value on $n_{yd} = \ln(\mu_{yd}/\mu_{y0})$ suggests that the net average gains that a young male expects from a match with an old female is lower than the average return from remaining unmarried. This does not mean that a young male would never marry an old female. On the contrary, realized marriages between young males and old females occur because of the large idiosyncratic payoffs drawn for these matches making this choice their utility maximizing option. These differences in net gains for young males across different types of spouse suggest that on average, marrying a young female is the most attractive choice while marrying an old female on average is less attractive than remaining single. Similar interpretations can be made with the remaining numbers in Table 3a and 3b.

Insert Table 3 here.

In Table 3c and 3d, the net gains by gender following the increase in the supply of young males from 26 to 36 are calculated. We use $n'_{ij}$ and $N'_{ij}$ to denote net gains in this alternate marriage market. Table 3e and 3f tabulates the change in the net gains resulting from this change in the supplies. Before the change, there is a total of 41 males and 47 females making males more scarce relative to females. This change increased the total number of single males to 51 while keeping the number females the same. As a consequence, females became relatively scarce. The sex ratio (that is the number of males per 100 females) increased from 87.23 to 108.51. The increase in the relative scarcity of females translates into an increase in the equilibrium transfers females receive in marriage. Since females are made more scarce following this change, more of the total output from marriage needs to be promised to females to ensure a match will occur. Table 3f) shows that the change in equilibrium transfers that females receive is positive for all matches, that is $\Delta \tau_{ij} = N'_{ij} - N_{ij} > 0$ for all $(i, j)$ pairs.

The changes in the net gains males receive, given in Table 3e tells an opposite story. Because this change made males less scarce, competition among single males for a spouse is more fierce. Males in this new marriage market equilibrium are willing to part with a larger portion of the total gains from marriage in order to find a spouse. Notice that the magnitude of the fall in average net gains for males is exactly equal (but opposite in
sign) to the magnitude of the increase in the net gains females expects on average, that is \((n'_{ij} - n_{ij}) = -(N'_{ij} - N_{ij})\) for all \((i,j)\) pairs. Since these changes in net gains within marriage cancel out, the average total gain from marriage relative to both parties remaining single remains unchanged. That is,

\[
(n'_{ij} + N'_{ij}) = (n_{ij} + N_{ij}) = 2\pi_{ij} \quad \forall \ i, j.
\tag{20}
\]

The numerical example highlights this feature of the CS model very clearly. To reiterate, changes in supplies and the sex ratio (which reflects changes in relative scarcity of males and females) affect the equilibrium transfer that clears the marriage market. This, in turn, alters the division of total average marital output between husbands and wives within marriage, but the total gains from marriage is not affected by changes in supplies. The precise nature of the changes in net gains however is not well understood. It currently remains a topic for future work.

**Adding Cohabitation**

Adding cohabitation to the CS model is straightforward. The setup of the model is similar to that discussed in the preceding section. We assume that there are three kinds of living arrangements denoted by \(l\). Each type \(i\) male (\(j\) female) can choose to remain unmarried. Otherwise, he or she can choose to find a spouse to marry \((l = m)\), or to cohabit with \((l = c)\). Each type \(i\) male (\(j\) female) may marry or cohabit with any type of female (male) subject to them being willing to pay (or accept) the transfer, \(\tau_{ij}^l\) associated with that living arrangement \(l\). Ignoring the choice to remain unmarried, each male (female) is now faced with \(2 \times J\) \((2 \times I)\) choices of living arrangements when choosing a spouse. This generates \(2 \times I \times J\) living arrangements for every combination of male and female. These \(2 \times I \times J\) submarkets clear when, given the equilibrium transfers \(\tau_{ij}^l\) associated with living arrangement \(l\), the demand for type \(j\) spouse in living arrangement \(l\) by \(i\) type males equals the supply of type \(j\) females matching with type \(i\) men in living arrangement \(l\). This condition has to hold for all \((i,j)\) pairs and all living arrangements \(l\).

The CS marriage matching function allowing for marriage and cohabitation takes the form,

\[
\ln \left[ \frac{\mu_{ij}}{\sqrt{\mu_{i0}\mu_{0j}}} \right] = \frac{\gamma_{ij}^l + \alpha_{ij}^l - (\alpha_{i0} + \gamma_{0j})}{2} = \pi_{ij}^l \quad \text{for } i \in \{m,c\},
\tag{21}
\]

where \(\mu_{ij}\) is the number of \(i, j\) couples in living arrangement \(l\), \(\mu_{i0}\) and \(\mu_{0j}\) are the number of
unmarried $i$ type males and $j$ types females respectively.\footnote{Note that the number of unmatched individuals includes individual who choses not to marry or cohabit. Mathematically,}

The parameters $\gamma_{ij}$ and $\alpha_{ij}$ are the common type $j$ female and type $i$ male contributions to output in living arrangement $l$, $\alpha_{i0}$ and $\gamma_{0j}$ are their separate common contributions while remaining single. Equation (21) states that the expected total gain per spouse from an $(i, j)$ match in living arrangement $l$ relative to the common gain from remaining single is a function of the number of such matches, $\mu_{ij}$ scaled by the geometric mean of the number of unmarrieds of these types, $\sqrt{\mu_{i0}\mu_{0j}}$. In other words, (21) states that the common gain for $(i, j)$ couples in living arrangement $l$ (relative to remaining single) is large when the number of such matches scaled by the geometric means of the number of unmarrieds is high.

Our extension to allow for cohabitation also identifies the common net gains for males and females in living arrangement $l$ analogously to (16) and (17). In equilibrium, the common net gains to type $i$ males from living arrangement $l$ with a type $j$ female is given by,

$$n_{ij}^l = \ln \left[ \frac{\mu_{ij}^l}{\mu_{i0}} \right] = \alpha_{ij}^l - \alpha_{i0} - \tau_{ij}^l,$$

where $l \in \{m, c\}$

The number of $(i, j, l)$ matches scaled by the number of $i$ type men who remain unmatched is a statistic for the common net gains to type $i$ males receive from such matches. Similarly the common net gains for type $j$ females in these matches is given by

$$N_{ij}^l = \ln \left[ \frac{\mu_{ij}^l}{\mu_{0j}} \right] = \gamma_{ij}^l - \gamma_{0j} + \tau_{ij}^l,$$

where $l \in \{m, c\}$.

The interpretation for the unmatched rates, $\ln(\frac{m_i}{\mu_{i0}})$ and $\ln(\frac{f_j}{\mu_{0j}})$ is straightforward and analogous to that provided earlier.

**Some Caveats**

First, the CS model is a static model of the marriage market. The marriage market is of course fundamentally dynamic. Single individuals who chosed to remain single this period may later choose to marry. Similar individuals who marry today could reenter the marriage market in the future in the event of divorce or the death of a spouse. Choo Siow (2005b) considers the complications arising from these dynamic concerns.

\footnote{Note that the number of unmatched individuals includes individual who choses not to marry or cohabit. Mathematically,}

$$\mu_{i0} = m_i - \sum_j \mu_{ij}^c - \sum_j \mu_{ij}^m,$$

and

$$\mu_{0j} = f_j - \sum_i \mu_{ij}^c - \sum_i \mu_{ij}^m$$
The current empirical application also considers age as the only characteristics that differentiate individuals in the marriage market. Our setup can be easily extended to allow for other characteristics.

Although the CS model has spillover effects and can fit any cross-section living arrangement distribution, the substitution effects are restrictive. At this point, they are not well understood.

**Marriage and cohabitation in contemporary Canada**

This paper estimates the systematic or expected gains to an \((i, j)\) couple marrying or cohabitating relative to remaining single in Canada in 1996. Our empirical analysis of Canadian data shows that the systematic gains to marriage increased with age and decreased with the age gap between spouses. The systematic gains to cohabitation is highest among young adults.

We use the estimated model to consider a number of interesting counterfactual experiments. In particular, we quantify the impacts of gender differences in mortality rates, and the arrival of the baby boom on the marriage and cohabitation distribution. The empirical methodology proposed is new to the literature.

As for gender differences in welfare, the higher mortality rate of men adversely affects women in the marriage market. However, this adverse effect is quantitatively modest because the gender differences in mortality rates is modest. The baby boom is a gender neutral but quantitatively large change in birth cohorts. Its estimated effect on the marriage market is quantitatively significant and not gender neutral. Older men derive substantial systematic net gains from being married to younger women. As of 1996, the baby boom made older individuals relatively scarce and middle age individuals relatively plentiful. So older men did not have to transfer as many resources to middle aged women to get them to marry them. The baby boom increased the net gains to entering the marriage market for older men and lowered the net gains for middle aged women.

The closest empirical paper to ours is Qian (1998) who used Schoen’s marriage matching function to study marriage and cohabitation in the US. His model did not allow for substitution effects. A comprehensive study of cohabitation and marriage in Canada, based on individual level data, is by Wu (2000).

**Data:** The data used were extracted from 1996 Canadian Census Public Use Microdata Files on Families and Individuals (PUMFF and PUMFI). A full description of the data extracted
is in the appendix. Both PUMFI and PUMFF files contain data based on a 2.8% sample of the population enumerated in the census. Since the age data in PUMFF was categorical, we used 6 age groups (15 to 24, 25 to 34, 35 to 44, 45 to 54, 55 to 64, 65 and over).

Insert Table 4 here.

Table 4 provides a summary of the data. The average age of females in the data is slightly higher than the average age of males reflecting the longer life expectancy of females. About 52% (50%) of males (females) 15 years and over were married and 8.3% (7.9%) of them were cohabitants. Thus, cohabitation was not a quantitatively significant choice for the general adult population. The average age of the married population is significantly higher than the average age of cohabitants. With a single cross section, we cannot tell if cohabitation is primarily a transitory phase among young adults or otherwise.

Figure 1 shows the numbers of males and females, the numbers of married males and married females, and the numbers of cohabitating males and cohabitating females by age. There are two features of the supply of individuals that are noteworthy. First, in every age group, there are more women than men. This difference is particularly large in the oldest age group. Second, the baby boom cohorts are substantially more numerous than the rest.

Figure 2 shows the systematic gains per partner in marriage. The systematic gains rise with age, and are highest along the age diagonal. The ridge along the age diagonal shows that the total gains fall faster for women married to younger men than for men married to younger women.

Figure 3 shows the systematic gains per partner in cohabitation. Unlike the systematic gains to marriage, the systematic gains to cohabitation is larger for younger individuals. The ridge along the age diagonal shows that the total gains fall faster for women cohabiting with younger men than for men cohabiting with younger women.

To show the difference in systematic gains between marriage and cohabitation, Figure 4 shows the systematic gains per partner in marriage and cohabitation for same aged couples. The systematic gains to marriage rise steeply with age, roughly flatten between age 35 to 60, and fall slowly after 60. Note that the systematic gains to marriage exceeded the gains to remaining single by age 35. The systematic gains to cohabitation are higher than that for marriage for young ages. It rises and peaks before age 30 and falls continuously thereafter. The systematic gains to cohabitation never exceed the gains to remaining single.

Another way to show the difference in systematic gains between marriage and cohabitation is in Figure 5. It shows the systematic gains to marriage minus the systematic gains to cohabitation. We divided the number of individuals in the first 5 age groups by 10 and the last age group by 25.
cohabitation. The difference in systematic gains for an $i, j$ pair is $\ln(\frac{\mu_{ij}}{\mu_{cij}})$, the log of the ratio of the number of marriages to the number of cohabitations. The differences in systematic gains increase with age as is already apparent in Figure 5. Also the differences in systematic gains falls rapidly off the age diagonal which says that for couples with wider age differences, the systematic gains to marriage fall relative to cohabitation. An explanation for this finding may be that the cost of separation is smaller under cohabitation and couples with wide age differences anticipate a smaller gain from remaining together in the future. While unsurprising, this finding has not been noticed before. It shows the value of our behavioral interpretation of the data.

Figure 6 shows the net gains to entering the marriage market for males and females. The net gains of a particular type is computed as the log of the inverse of the share of that type that remained single. The net gains increase when the share of a type remaining single falls. What is striking in Figure 6 is the gender differences in net gains by age. The net gains of women peaked around age 40 whereas that for men, they peak substantially later. The net gains to entering the marriage market for individuals of a particular type depend on both preference parameters as well as equilibrium transfers that these individuals receive. Equilibrium transfers are affected in part by population supply considerations. We use our model to see how the net gains are affected by two different experiments.

**Effects of Differential Mortality Rates**

As seen in Figure 1, men have higher mortality rates than women. So can part of the lifecycle gender differences in net gains be explained by the gender difference in mortality rates? We answer this question by constructing counterfactual male and female population vectors, $M'$ and $F'$, where

$$M' = F' = \frac{M + F}{2}$$

Using our estimated systematic gains to marriage and cohabitation (which are invariant to changes in supplies), we make predictions of the distribution of marriages and cohabitation under this counterfactual population vectors.

The observed and predicted quantities by gender and different age groups are given in Table 5. The initial and new population vectors (with no gender differences in mortality) for males and females are tabulated in columns 2 and 3 of Table 5a and 5b. Columns 4 and 5 gives the observed and predicted marriages. Observed and predicted number of cohabitations are given in columns 6 and 7. As expected, male population has increased whereas the female population has decreased. However the changes in populations are not large except for the
oldest age group. The CS model predicts that the number of marriages and cohabitations would increase by 63177 and 2849 respectively.\(^{19}\)

Comparing the number of marriages and cohabitations across age groups, the changes between predicted and observed are small. Marriages for males younger than 64 is predicted to fall slightly while cohabitation for this same group is predicted to rise. These predicted changes are of magnitudes less than 1 percent of the level in 1996. The largest change is predicted for males aged 65 and older. Removing gender differences in mortality increases male 65 and older by around 16 percent. This is predicted to raise marriages and cohabitations for this group by around 7 and 11 percent respectively.

Despite a decrease in the total number of single women by around 3 percent, marriages and cohabitations for women is predicted to increase. The magnitude of the predicted changes for females is larger than that for males. Marriages and cohabitation for women 24 and younger, and 45 and older is predicted to increase by more than one percent. The largest increase is predicted for women aged 55 and older.

The model also makes predictions on changes to net gains from entering the marriage market. Figure 7 compares the original net gains and the net gains under the counterfactual population vectors. As expected, the new net gains for women increased and that for men decreased. However, the age at which the female net gains falls below the male net gains only increases slightly. Thus, for a modest change in population vectors, there is a modest change in net gains. The higher mortality rate of men makes men more scarce relative to women. We show that this modestly reduced (increased) the welfare of women (men) in the marriage market. While our interpretation of Figure 7 is behavioral, one can use a purely descriptive interpretation. The CS marriage matching function predicts that the fraction of males remaining single will rise whereas the fraction of females remaining single will fall if gender differences in mortality are erased. This is given in Table 5c.

**Effects of the Baby Boom**

The second counterfactual experiment we consider is to remove the baby boomers from the marriage market. Instead of having birth and immigration cohorts of different sizes, consider a population with equal sized birth cohorts and no immigration. In this case, the stationary lifecycle population vectors are determined completely by mortality rates. We use the Statistic Canada 2002 life tables to determine the shape of the the stationary population vectors, \(M^*\) and \(F^*\). We matched the total population by gender to get the levels for \(M^*\) and

\(^{19}\)This accounts for a modest 1.1% and 0.32 % increase in marriages and cohabitations respectively.
Figure 8 shows the 1996 population vectors, $M$ and $F$, and the stationary population vectors, $M^*$ and $F^*$.

The areas under $M$ and $M^*$ are the same. Likewise for $F$ and $F^*$. The stationary population vectors are substantially different from the 1996 population vectors. In particular, there are many fewer middle aged individuals and many more older individuals in the stationary population vectors. Removing the baby boomers is quantitatively a much larger change in population vectors than equalizing gender mortality rates.

Table 6a and 6b gives the observed and predicted quantities by gender and age groups from this counterfactual experiment. These changes are expected to generate around 116,088 more marriages, a modest increase of around 2 percent. Although the increase in total marriages is small, there are substantial changes across age groups. Marriages by males and females aged 44 and younger is predicted to decrease by 20 and 17 percent respectively. While marriages by males and females of age 45 and older is predicted to increase by 43 and 51 percent respectively.

As is already apparent in Figure 1 and confirmed in Figures 4 and 5, cohabitation is more prevalent among young adults. Thus, one would expect that removing the baby boomers would result in a substantial decline in cohabitations. Figure 9 shows the log of the ratio of number of cohabitants by gender in 1996 relative to the number predicted for the stationary population. As expected, the baby boomers substantially increase the number of male and female cohabitants under the age of 40. These changes are also shown in columns 6 and 7 of Table 6a and 6b. Removing the baby boomers is predicted to lower the total number of cohabitations by 14 percent. While the number of male and female cohabitants under the age of 44 fell, cohabitation among individuals 45 and older is predicted to increase slightly.

The impact of the baby boomers on net gains is also quantitatively substantial. Figure 10 shows the net gains for the 1996 population and the stationary population. Essentially men of all ages benefitted from the baby boom whereas women did not. The decrease in net gains for women due to the baby boom is concentrated between the ages of 30 and 60. The increase in net gains for men occurs primarily after forty and persists for the remainder of life. An explanation for this gender difference in net gains is as follows. Older men derive substantial systematic gross gains from being married to younger women. The baby boom made older individuals relatively scarce and middle age individuals relatively plentiful. This advantages older men in the marriage market by reducing equilibrium transfers. Thus, the baby boom increased the net gains to older men and lowered the net gains of middle aged

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20We divided the number of individuals in the first 5 age groups by 10 and the last age group by 25.

21Columns 2 and 3 of the table provides the population vector graphed in Figure 8. Notice that the total number of single men and women are kept the same in this experiment.
women. To provide evidence for this hypothesis, Figure 11 plots the statistic $\eta_i$ against the age of men where:

$$\eta_i = \ln\left(\frac{\mu_{m_i,39.3} + \mu_{c_i,39.3}}{\sum_k \mu_{m_k,39.3} + \mu_{c_k,39.3}}\right) - \ln\left(\frac{\mu_{m^*_i,39.3} + \mu_{c^*_i,39.3}}{\sum_k \mu_{m^*_k,39.3} + \mu_{c^*_k,39.3}}\right).$$

$\eta_i$ is the log of the ratio of the share of age $i$ men in coupled living arrangements with women in the 39 age group in 1996 relative to the share in a stationary population. $\eta_i$ is increasing in $i$ after age 39. What this means is that in 1996, the share of older men in coupled living arrangements with 39 year old women is larger than the share of older men with the same aged women in a stationary population. While the baby boom is a gender neutral change in birth cohorts, its estimated effect on the marriage market is not gender neutral.

Even after accounting for gender differences in mortality and baby boom effects, there are significant lifecycle gender differences in the net gains to entering the marriage market. Siow (1998) provides an explanation based on the behavioral consequences of gender differences in fecundity.

**Conclusion**

We used a transferable utility model of the marriage market to rationalize marriage and cohabitation behavior in contemporary Canada. The estimated model is used to quantify the impacts of gender differences in mortality rates and the baby boom on observed behavior. The effect of gender differences in mortality on gender differences in net gains from entering the marriage market is quantitatively modest and in the expected direction. The baby boom is a gender neutral but quantitative large change in birth cohorts. Its estimated effect on the marriage market is quantitatively significant and not gender neutral.

Although the CS model is a structural model of the marriage market, it is non-parametric and fits the 1996 data exactly. Thus the CS model can be used descriptively without behavioral interpretation. An advantage of the behavioral interpretation is that we can use it to think about gender differences in welfare in the marriage market without having to estimate male or female preferences.

Much remains to be done. The substitution properties of the model is not well understood. The stability and determinants of the systematic gains to marriage have to be investigated. We have barely begun to investigate the empirical usefulness of this model.
DATA APPENDIX

The data used were extracted from 1996 Canadian Census Public Use Microdata Files on Families and Individuals (PUMFF and PUMFI). These files provide information on the demographic, social and economic characteristics of the Canadian Population (family and non-family persons). Both PUMFI and PUMFF files contain data based on a 2.8% sample of the population enumerated in the census. The target population in the Family file includes all census families composed of Canadian citizens, landed immigrants and non-permanent residents living in a private dwelling on Census Day; the target population in the Individual file includes all Canadian citizens, landed immigrants and non-permanent residents having a usual place of residence in Canada.

Although both files contain data about family status of the individuals, the records in the Family file keep track of the spouses and common-law partners of heads of the households. The data in Individual file have no serial numbers and no data pertaining to the persons’ spouses or common-law partners. Therefore, in the calculations involving married and cohabiting couples, we only used data from PUMFF. A cohabitating couple is defined as follows. Question 6 in the individual census form asks:

“Is this person living with a common-law partner? (Common-law refers to two people who live together as husband and wife but who are not legally married to each other.)”

If the answer is yes, the individual and his or her partner were coded as cohabitating by the census.

The age range studied was from 15 to 75 and over. However, the age variable in the Family file, denoted as ‘agef’ for female persons and ‘agem’ for male persons, was compressed into 7 age categories (15 to 24, 25 to 34, 35 to 44, 45 to 54, 55 to 64, 65 to 74, 75 and over), while the age variable in the Individual file, denoted as ‘agep’ refers to the age at last birthday. To make these variables consistent, we compressed the age variables in both PUMFI and PUMFF into 7 age categories (15 to 24, 25 to 34, 35 to 44, 45 to 54, 55 to 64, 65 to 74, 75 and over).

We use ‘cfstruc’ (Census Family Structure) variable from PUMFF file to determine whether a family falls into one of these categories: family of a now-married couple with or without never married children of either or both spouses, family of a common-law couple with or without children of either or both spouses, or a lone-parent family. Because we are only interested in observations containing married and common-law couples with or without
children, we remove the rest of unnecessary records. We then separate the files into two datasets, one containing age distributions of married couples, and the other one containing age distributions of common-law couples. This process did not present complexity, and therefore we only lost a small number of observations.

Calculating the number of available persons was not as straightforward, because PUMFI contains records of both family and available persons. The PUMFI file does not contain the 'sfstruc' variable we mentioned earlier, so the layout and distribution of records' family status in PUMFI are slightly different than those in the PUMFF file. Therefore, in order to determine the number of available males, we subtracted the sum of married and common-law couples from a total number of males in PUMFI. Derivation of available females was analogous. The differences between the numbers of available persons of both sexes that we have calculated and the actual numbers from the PUMFI follow further below in the document.

Some of the discrepancies in our calculations have arisen due to the unavailable or missing data in the records. Whenever we encountered a missing value pertaining to the variable we required in our calculations, we were forced to delete the entire record.

As mentioned above, we have lost a number of observations during our extractions of data for married, cohabiting and available persons. The numbers for actual and observed calculations follow below:

- Total number of observations in the PUMFF extract file: 342,231.
- Number of observations of married couples with or without children in PUMFF extract file: 161,315.
- Number of observations of married couples with or without children after we have deleted missing age variables: 161,295.  
- Number of observations of common-law couples with or without children in PUMFF extract file: 25,384.
- Number of observations of common-law couples with or without children after we have deleted missing variables: 25,380.  
- Total number of observations of males in PUMFI extract file: 389,113

---

22 We deleted 8 observations due to missing ‘agef’ (age of female partner) and 12 observations due to missing ‘agem’ (age of male partner).
23 We deleted 2 observations due to missing ‘agef’, and 2 observations due to missing ‘agem’.
• Number of observations of available males after we have applied our extraction method and deleted all missing variables: 117849

• Total number of observations of females in PUMFI extract file: 403 335

• Number of observations of available females after we have applied our extraction method and deleted all missing variables: 135 978

• Total number of observations deleted due to missing age variable in PUMFI: 58
References


Hamilton, G. and A. Siow. 2000. “Class, Gender and Marriage”, University of Toronto manuscript available at “www.economics.utoronto.ca/siow”.


Table 1: Marriages and supplies in a marriage market with two types of males and females

<table>
<thead>
<tr>
<th></th>
<th>WOMEN</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YOUNG</td>
<td>OLD</td>
</tr>
<tr>
<td>women</td>
<td>$f_y = 30, \mu_{0y} = 8$</td>
<td>$f_d = 17, \mu_{0d} = 6$</td>
</tr>
<tr>
<td>men</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YOUNG</td>
<td>$m_y = 26$</td>
<td>$\mu_{yy} = 16$</td>
</tr>
<tr>
<td></td>
<td>$\mu_{y0} = 6$</td>
<td></td>
</tr>
<tr>
<td>OLD</td>
<td>$m_d = 15$</td>
<td>$\mu_{dy} = 6$</td>
</tr>
<tr>
<td></td>
<td>$\mu_{d0} = 2$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Predicted marriages from increase in number of young males

<table>
<thead>
<tr>
<th></th>
<th>a) Schoen matching function</th>
<th>b) CS matching function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>young female</td>
<td>old female</td>
</tr>
<tr>
<td></td>
<td>$f_y = 30$</td>
<td>$f_d = 17$</td>
</tr>
<tr>
<td></td>
<td>$\mu_{0y} = 5.203$</td>
<td>$\mu_{0d} = 5.507$</td>
</tr>
<tr>
<td>young male</td>
<td>$m_y = 36$</td>
<td>18.797</td>
</tr>
<tr>
<td></td>
<td>$\mu_{y0} = 12.709$</td>
<td></td>
</tr>
<tr>
<td>old male</td>
<td>$m_d = 15$</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>$\mu_{d0} = 2$</td>
<td></td>
</tr>
</tbody>
</table>
### Table 3: Changes in net common gains to marriage for numerical example

<table>
<thead>
<tr>
<th>Net common gains by gender before supply change</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a) Male's</strong></td>
<td><strong>b) Female's</strong></td>
</tr>
<tr>
<td>$n_{ij}$</td>
<td>$N_{ij}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>young male $m_y = 26$</td>
<td>young male $m_y = 26$</td>
</tr>
<tr>
<td>$n_{yy} = 0.9808$</td>
<td>$N_{yy} = 0.6931$</td>
</tr>
<tr>
<td>$n_{yd} = -0.4055$</td>
<td>$N_{yd} = -0.4055$</td>
</tr>
<tr>
<td>old male $m_d = 15$</td>
<td>old male $m_d = 15$</td>
</tr>
<tr>
<td>$n_{dy} = 1.0986$</td>
<td>$N_{dy} = -0.2877$</td>
</tr>
<tr>
<td>$n_{dd} = 1.2528$</td>
<td>$N_{dd} = 0.1542$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Net common gains by gender after supply change</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>c) Male's</strong></td>
<td><strong>d) Female's</strong></td>
</tr>
<tr>
<td>$n'_{ij}$</td>
<td>$N'_{ij}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>young male $m'_y = 36$</td>
<td>young male $m'_y = 36$</td>
</tr>
<tr>
<td>$n'_{yy} = 0.4529$</td>
<td>$N'_{yy} = 1.2210$</td>
</tr>
<tr>
<td>$n'_{yd} = -0.8545$</td>
<td>$N'_{yd} = 0.0436$</td>
</tr>
<tr>
<td>old male $m'_d = 15$</td>
<td>old male $m'_d = 15$</td>
</tr>
<tr>
<td>$n'_{dy} = 0.8152$</td>
<td>$N'_{dy} = -0.0043$</td>
</tr>
<tr>
<td>$n'_{dd} = 1.0483$</td>
<td>$N'_{dd} = 0.3586$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Change in net common gains by gender</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>e) Male's</strong></td>
<td><strong>f) Female's</strong></td>
</tr>
<tr>
<td>$(n'<em>{ij} - n</em>{ij})$</td>
<td>$(N'<em>{ij} - N</em>{ij})$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>young male</td>
<td>young male</td>
</tr>
<tr>
<td>$n'<em>{yy} - n</em>{yy} = -0.5279$</td>
<td>$N'<em>{yy} - N</em>{yy} = 0.5279$</td>
</tr>
<tr>
<td>$n'<em>{yd} - n</em>{yd} = -0.4490$</td>
<td>$N'<em>{yd} - N</em>{yd} = 0.4490$</td>
</tr>
<tr>
<td>old male</td>
<td>old male</td>
</tr>
<tr>
<td>$n'<em>{dy} - n</em>{dy} = -0.2833$</td>
<td>$N'<em>{dy} - N</em>{dy} = 0.2833$</td>
</tr>
<tr>
<td>$n'<em>{dd} - n</em>{dd} = -0.2045$</td>
<td>$N'<em>{dd} - N</em>{dd} = 0.2045$</td>
</tr>
</tbody>
</table>
Table 4: Data Summary

<table>
<thead>
<tr>
<th></th>
<th>Number of ind. (millions)</th>
<th>Mean Age</th>
<th>S.D. of age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total males</td>
<td>10.87</td>
<td>34.5</td>
<td>21.1</td>
</tr>
<tr>
<td>Total females</td>
<td>11.52</td>
<td>36.2</td>
<td>21.8</td>
</tr>
<tr>
<td>Married male</td>
<td>5.76</td>
<td>49.4</td>
<td>14.5</td>
</tr>
<tr>
<td>Married female</td>
<td>5.76</td>
<td>46.7</td>
<td>14.1</td>
</tr>
<tr>
<td>Cohabiting male</td>
<td>0.906</td>
<td>37.5</td>
<td>11.7</td>
</tr>
<tr>
<td>Cohabiting female</td>
<td>0.906</td>
<td>34.8</td>
<td>11.1</td>
</tr>
</tbody>
</table>
Table 5: Comparing Observed and Predicted Quantities when Mortality Rates across Gender Are the Same

### a) Marriages and Cohabitation for Males

<table>
<thead>
<tr>
<th>Age groups</th>
<th>Single Males</th>
<th>Marriages</th>
<th>Cohabitation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>observed</td>
<td>counterfactual</td>
<td>observed</td>
</tr>
<tr>
<td>15 to 24</td>
<td>1907787.7</td>
<td>1896430.6</td>
<td>52287.7</td>
</tr>
<tr>
<td>25 to 34</td>
<td>2165857.1</td>
<td>2205946.4</td>
<td>893857.2</td>
</tr>
<tr>
<td>35 to 44</td>
<td>2361535.8</td>
<td>2405285.8</td>
<td>1520571.5</td>
</tr>
<tr>
<td>45 to 54</td>
<td>1832714.3</td>
<td>1846196.5</td>
<td>1321928.6</td>
</tr>
<tr>
<td>55 to 64</td>
<td>1209500.0</td>
<td>1226626.0</td>
<td>928357.1</td>
</tr>
<tr>
<td>65 &amp; older</td>
<td>1398466.3</td>
<td>1619108.2</td>
<td>1043535.7</td>
</tr>
<tr>
<td>Total</td>
<td>10875861.2</td>
<td>11199593.4</td>
<td>5760537.8</td>
</tr>
</tbody>
</table>

### b) Marriages and Cohabitation for Females

<table>
<thead>
<tr>
<th>Age groups</th>
<th>Single Females</th>
<th>Marriages</th>
<th>Cohabitation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>observed</td>
<td>counterfactual</td>
<td>observed</td>
</tr>
<tr>
<td>15 to 24</td>
<td>1885073.4</td>
<td>1896430.6</td>
<td>127035.7</td>
</tr>
<tr>
<td>25 to 34</td>
<td>2246035.7</td>
<td>2205946.4</td>
<td>1138785.7</td>
</tr>
<tr>
<td>35 to 44</td>
<td>2449035.8</td>
<td>2405285.8</td>
<td>1593964.4</td>
</tr>
<tr>
<td>45 to 54</td>
<td>1859678.6</td>
<td>1846196.5</td>
<td>1270000.0</td>
</tr>
<tr>
<td>55 to 64</td>
<td>1243752.0</td>
<td>1226626.0</td>
<td>845930.6</td>
</tr>
<tr>
<td>65 &amp; older</td>
<td>1839750.0</td>
<td>1619108.2</td>
<td>784821.5</td>
</tr>
<tr>
<td>Total</td>
<td>11523325.5</td>
<td>11199593.4</td>
<td>5760537.8</td>
</tr>
</tbody>
</table>

### c) Number of Unmarrieds as percentage of singles

<table>
<thead>
<tr>
<th>Age groups</th>
<th>Unmarried Males (%)</th>
<th>Unmarried Females(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>observed</td>
<td>predicted</td>
</tr>
<tr>
<td>15 to 24</td>
<td>92.35</td>
<td>92.36</td>
</tr>
<tr>
<td>25 to 34</td>
<td>43.03</td>
<td>44.14</td>
</tr>
<tr>
<td>35 to 44</td>
<td>25.06</td>
<td>26.28</td>
</tr>
<tr>
<td>45 to 54</td>
<td>20.26</td>
<td>21.19</td>
</tr>
<tr>
<td>55 to 64</td>
<td>18.51</td>
<td>20.03</td>
</tr>
<tr>
<td>65 &amp; older</td>
<td>23.46</td>
<td>29.18</td>
</tr>
</tbody>
</table>
Table 6: Comparing Observed and Predicted quantities when baby boomers are not present

**a) Marriages and Cohabitation for Males**

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>Single Males</th>
<th>Marriages</th>
<th>Cohabitation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Predicted</td>
<td>Observed</td>
</tr>
<tr>
<td>15 to 24</td>
<td>1907787.7</td>
<td>1787002.7</td>
<td>52287.7</td>
</tr>
<tr>
<td>25 to 34</td>
<td>2165857.1</td>
<td>1769078.6</td>
<td>893857.1</td>
</tr>
<tr>
<td>35 to 44</td>
<td>2361535.8</td>
<td>1743879.3</td>
<td>1520571.5</td>
</tr>
<tr>
<td>45 to 54</td>
<td>1832714.3</td>
<td>1697152.6</td>
<td>1321928.6</td>
</tr>
<tr>
<td>55 to 64</td>
<td>1209500.0</td>
<td>1583459.4</td>
<td>928357.1</td>
</tr>
<tr>
<td>65 and older</td>
<td>1398466.3</td>
<td>2295288.7</td>
<td>1043535.7</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10875861.2</strong></td>
<td><strong>10875861.2</strong></td>
<td><strong>5760537.8</strong></td>
</tr>
</tbody>
</table>

**b) Marriages and Cohabitation for Females**

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>Single Females</th>
<th>Marriages</th>
<th>Cohabitation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Predicted</td>
<td>Observed</td>
</tr>
<tr>
<td>15 to 24</td>
<td>1885073.4</td>
<td>1757161.7</td>
<td>127035.7</td>
</tr>
<tr>
<td>25 to 34</td>
<td>2246035.7</td>
<td>1750741.8</td>
<td>1138785.7</td>
</tr>
<tr>
<td>35 to 44</td>
<td>2449035.8</td>
<td>1739178.8</td>
<td>1593964.4</td>
</tr>
<tr>
<td>45 to 54</td>
<td>1859678.6</td>
<td>1711168.7</td>
<td>1270000.0</td>
</tr>
<tr>
<td>55 to 64</td>
<td>1243752.0</td>
<td>1641507.2</td>
<td>845930.6</td>
</tr>
<tr>
<td>65 and older</td>
<td>1839750.0</td>
<td>2923567.4</td>
<td>784821.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>11523325.5</strong></td>
<td><strong>11523325.5</strong></td>
<td><strong>5760537.8</strong></td>
</tr>
</tbody>
</table>
Figure 1 - Men and Women in Canada by age, 1996

Figure 2 - Common Total Gains from marriage, $\pi_{ij}^m$
Figure 3 - Common Total Gains from cohabitation, $\pi_{ij}$

Figure 4 - Common Total Gains for same aged couples
Figure 5 - Difference in common total gains, $\pi_{ij}^{m} - \pi_{ij}^{c}$

Figure 6 - Net Gains by age
Figure 7 - Net Gains when mortality rates are the same

Figure 8 - Total men and women with and without baby boom
Figure 9 - Impact of the baby boom on cohabitation

Figure 10 - Net Gains with and without baby boom
Figure 11 - Impact of the baby boom on cohabitation