Monogamy implies positive assortative matching

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Abstract

Becker’s transferable utility model of a frictionless marriage market is used to analyze the determination of marital regimes, monogamy versus polygyny. There is positive assortative matching by spousal abilities in almost all monogamous societies. Whether or not such societies also practice the sexual division of labor is irrelevant for predicting its marriage matching pattern. The necessary condition for equilibrium monogamy is stronger than Becker’s classic complementarity condition. Polygyny occurs when monogamy is not an equilibrium. Different marriage matching patterns occur within polygyny.

In two landmark contributions, Becker introduced the transferable utility model of a frictionless marriage market. He used it to analyze monogamy and polygamy, and assortative matching by spousal abilities under monogamy. While he discussed the circumstances which determine monogamy versus polygamy, he did not fully analyze the determination of marital regimes. The objective of this paper is to use his framework to analyze the determination of marital regimes, monogamy versus polygyny, and the resulting assortative matching patterns.

Unlike Becker, this paper ignores polyandry where women marry multiple husbands simultaneously. The rationale for ignoring polyandry is due to gender differences in fecundity. Women can have as many children as they want with

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2Other implications of differential fecundity are explored in Ackerlof, et. al. (1900), Buckles (2005), Erosa, et. al. (2005), Francesconi (2002), Giolito (2003), Hamilton and Siow, Siow (1998), Siow (2002).
a single husband. On the other hand, a man is limited in the number of children he can have with one wife. He can have more children if he has more wives. As shown in the literature review below, this motivation for understanding polygyny is standard. It also accords with the historical and contemporary record that polygyny is common whereas polyandry is rare (E.g. Murdock, 1981).

The analysis in this paper sheds light on an empirical anomaly when Becker’s theory of marriage matching is considered together with his theory on the sexual division of labor. With transferable utility, a sufficient condition to obtain positive assortative matching in marriage by ability in monogamous societies is that the abilities of the spouses are complementary in producing marital output.\footnote{If marital output is differentiable in abilities, the cross partials of abilities are positive; or marital output is supermodular in abilities. Modern statements are in Legros and Newman (2002) and Weiss (1997).} When the abilities of the spouses are substitutes in producing marital output,\footnote{If marital output is differentiable in abilities, the cross partials of abilities are negative; or marital output is submodular in abilities.} negative assortative matching in marriage is an equilibrium. Becker’s theory on the sexual division of labor argued that one role of marriage is that spouses can specialize in activities within the household and realize gains from specialization. Assume that it is efficient for one spouse to work in the labor market and the other spouse to engage in home production. Then the principle of comparative advantage assigns the spouse that is relatively more productive in the labor market to do market work and vice versa. When men are relatively more productive in the labor market than women, Becker’s insight rationalizes the traditional sexual division of labor where the husband works in the labor market and the wife engages in home production.

When Becker’s two contributions are considered together, an empirical anomaly arises.\footnote{This discussion follows Weiss 1997.} Household specialization suggests that marital output is an increasing function of \( \max(W_m, W_f) \) where \( W_m \) is the wage of the husband and \( W_f \) is the wage of the wife. Unless the wages of the spouses are equal, the cross partial of marital output with respect to the spouses’ abilities is zero. That is, spousal wages are not complements in producing marital output. In this case, Becker’s contribution on marital matching suggests that spouses should sort negatively by wages. The intuition is clear. When only one spouse works, it is not efficient for two high wage spouse to marry each other. The labor market productivity of the non-working high wage spouse is unused. If the variation in home production productivities are small relative to productivities in the labor market, negative assortative marriage matching by labor market wages is the equilibrium outcome. The empirical anomaly is that negative assortative matching by labor market productivity is not observed. Positive assortative matching by education, wages and parental resources is ubiquitous in monogamous societies (E.g. Fafchamps and Quisumbing (2005), Fernandez, et. al. (2005), Hamilton and Siow (forthcoming), Qian and Preston (1993), Suen and Lui (1999)).\footnote{With non-transferable utilities, positive assortative matching by spousal abilities obtain as long as marital output are increasing in spousal abilities (E.g. Lam (1988), Peters and Siow (2004)). Retaining transferable utilities, assume labor market productivity is highly}
The anomaly arises when monogamy is assumed. This paper requires that an observed marital regime is an equilibrium phenomena. In this case, almost all equilibrium monogamous societies will exhibit positive assortative matching in marriage by spousal abilities. Whether or not a monogamous society also practice the sexual division of labor is irrelevant for predicting its marriage matching pattern.

In fact Becker’s sufficient condition for positive assortative matching is not sufficient when individuals can choose to be polygynous. A stronger necessary condition is needed. Put another way, when complementarity fails, monogamy is not an equilibrium outcome. Polygyny is the equilibrium outcome. The intuition is as follows. When complementarity fails, high ability individuals will receive large rents from marrying down. Men who earn ability rents will want to marry more than one wife. They can do so by marrying down. More subtly, if mix ability marriages with high ability wives are productive, low ability men may want to marry up. Thus one may observe negative assortative matching in polygyny, some low ability men marrying up and some high ability men marrying down polygynously. Monogamy occurs only when there is a much stronger form of complementarity whereby the high ability spouses produce much higher levels of output with each other than with lower ability spouses. Thus there is essentially one marriage matching pattern under monogamy. There are different marriage matching patterns under polygyny, depending on parameter values. An empirically important prediction of the model is that the set of parameters which support monogamy do not overlap with that which support polygyny.

Interestingly, although perfect negative assortative matching under polygyny is feasible, it does not exists in equilibrium. Thus a conclusion of this paper is that perfect negative assortative matching does not exists under monogamy nor polygyny.

The model considered here is stylized. Its primary virtue is that it uses Becker’s framework, which the current literature on assortative marital matching under monogamy builds on, to lay out a framework for studying assortative marital matching and equilibrium marital regimes.

1 Literature review

Readers who are primarily interested in assortative marital matching and equilibrium marital regimes can read the model first and then return to the literature review. For completeness, I also survey one sex heterogeneity models which are not directly related to this paper.

Gould, Moav and Simhon’s (2003) provides the first model of equilibrium marital regimes with two sex heterogeneity. Using transferable utilities and a frictionless marriage market, they study the interaction between adult het-

correlated with productivity in home production and the variation in productivity at home is large (E.g. Becker, Weiss). In other words, complementarity of spousal abilities, as proxied by education or wages, in marital production is preserved. Both responses rationalize the anomaly at the cost of weakening the predictive power of Becker’s contributions.
erogeneity and parental investments to understand why modern societies are monogamous. This paper follows their demographic structure. Due to their specific concerns, their marital output function is more restrictive and therefore their model reproduces a subset of the results derived here. Our papers are complementary. I focus on how the structure of marital output affects marital regimes. They are focussed on specific household processes which may lead to monogamy in modern societies.

There is a long tradition of models on equilibrium marital structures which ignores female heterogeneity. The benchmark model is the “polygyny threshold” model (Emlen and Oring 1977; Orians 1969). The “polygyny threshold” model is a one factor model of male heterogeneity.\(^7\) Within a society, all women are homogenous. Men differ in one dimension, their access to resources. Men prefer more wives to less. An evolutionary argument for this preference is that a man can have more children if they have more wives. Children per mother is limited by the long gestation period that a woman needs to have a child and menopause. Children per father is only limited by his access to mothers.

Assuming marriage market clearing, since all women are the same, they must each get the same reservation utility in marriage. Assume that all women marry. Then this reservation utility is larger than that from remaining single. If a man has enough resources to provide the reservation level of utility to two women, he will be able to marry two wives. Men who do not have enough resources to provide the reservation level of utility to a woman will have no wife. These unmarried men are outbid for wives by their richer monogamous and polygynous peers. The key prediction of this model is that resource inequality among men within a society is central to generating polygyny. If there is a low degree of resource inequality among men in the society, the model will predict a low degree of polygyny because few men can afford multiple wives. Social scientists have found some empirical support for this model.\(^8\) A problem with this benchmark model is that it cannot explain the prevalence of monogamy in modern industrial societies. Presumably the wealthiest men in these societies can outbid poorer men for multiple wives if they so desire.\(^9\)

Researchers have extended the benchmark model to differentiate between effective resource and gross resource. The utility that a wife obtains depends on the amount of effective resource that she gets. There is a societal specific production function which transforms gross resource of the husband into his total effective resource. The effective resource per wife is his total effective resource divided by the number of wives.

Let the production function of total effective resource of a male in a particular society, \(r_e\), be:

\[
r_e = \alpha r_g^\beta ; \quad \beta, \alpha > 0
\]

(1)

where \(r_g\) is his total gross resource. \(\alpha\) and \(\beta\) are specific to that particular

\(^7\)Betzig (1997) is a collection of applications to human societies.
\(^8\)E.g. Betzig, Kaplan and Lancaster 2002.
\(^9\)Researchers have found polygyny practiced in societies where the resource inequality among men does not seem large by contemporary standards (E.g. Borgerhoff Mulder (1989, 1990), Jacoby (1995)).
society. Different authors present different structural models for $\alpha$ and $\beta$. Since women care about effective resource and not gross resource, it is inequality in effective resource that determine the degree of polygyny. A simple way to see how $\beta$ determines effective resource inequality is to take logs of (1) to get:

$$\ln r_e = \ln \alpha + \beta \ln r_g$$

A measure of effective resource inequality in the society is to consider the variance of $\ln r_e$ in that society:

$$\text{var}(\ln r_e) = \beta^2 \text{var}(\ln r_g)$$

Two societies, with the same degree of gross resource inequality, $\text{var}(\ln r_g)$, will have different degrees of effective resource inequality if they have different values for $\beta$. Effective resource inequality and therefore the degree of polygyny is increasing in $\beta$. Although $\alpha$ affects the average level of welfare of the society, it does not in general have an effect on the degree of polygyny.

The benchmark “polygyny threshold” model implicitly assumes that $\beta = 1$ for all societies. Therefore effective resource is gross resource and only societal resource inequality determine the degree of polygyny.

A particular model of why $\beta$ differ across societies is Boserup (1970). She argues that different agricultural techniques, which differentially affect the productivities of wives in agriculture in Africa and Asia, can explain the differences in marital systems between the two continents. Jacoby (1995) provides empirical support for her model.

In Becker’s model of marital systems, $\beta$ depends on the relative contributions of paternal versus maternal investment in children. He suggests that $\beta$ has declined in modern industrial societies due to the increased productivity of paternal investment in time relative to maternal investment in time (Becker 1991, p. 94-95). So he argues that the degree of polygyny should be lower in modern industrial societies (Also see Marlowe (2000)).

However Becker’s classic quantity and quality of children model implies that modern societies, where labor earnings are more important that non-labor earnings in determining male resources, should be polygynous. High wage fathers, who have a high cost of time, should economize on paternal time (investing in the quality of children). Instead, they should substitute toward a larger quantity of low quality children (polygyny).\(^{10}\)


\(^{10}\)In an unpublished study with Jeanne LaFortune, we investigated this model. We obtained monogamy for high wage men by assuming that each child required a sufficiently high fixed cost of paternal time. We will relate this model to the one in this paper in sub-section 12.1. Also see Gould, et. al.
2 Model

Consider a stationary society where the sex ratio, the ratio of adult males to adult females, is one. I normalize the population of each gender to one. Men and women are differentiated by their social status $m$ and $f$ respectively. There are two social statuses, $m = \{h, l\}$ and $f = \{h, l\}$, $h > l$. Let the fractions of high status men and women be both equal to $\alpha$ and the fractions of low status men and women be both equal to $1 - \alpha$. Assume that $\alpha < \frac{1}{2}$.

Building on Becker (1991; p. 124), let $\pi_{mf}n^\beta$ be total household output of a husband with social status $m$, and $n$ wives, each with social status $f$. As discussed in the introduction and the literature review, the marital output production function used here is motivated by the assumption that the main reason for marriage is to have children, and men can have more children if they have more spouses but not vice versa.

For convenience, assume that all wives in a household have the same social status. There is decreasing returns in wives. $\frac{1}{2} < \beta < 1$. If $\beta$ is too small, there is little gain to polygyny.

Also for convenience, $n$ is continuous. The interpretation of an optimal $n$ when it is not an integer is as follows. When $p < n < p + 1$ for non-negative integer $p$, some type $m$ men have $p$ wives and others have $p + 1$. The average number of wives per type $m$ man is $n$. Thus observationally, all men have integer number of wives. When $n$ is a fraction, some type $m$ men will be observed to be unmarried.

The productivity of different types of marital matches in producing household output is governed by $\pi_{mf}$. It satisfies:

\[
\begin{align*}
\pi_{ll} &= 1 \\
\pi_{ll} \leq \pi_{lh} \leq \pi_{hh} \\
\pi_{ll} \leq \pi_{lh} \leq \pi_{hh}
\end{align*}
\]

$\pi_{ll} = 1$ is a normalization.

Let $\tau_f^X$ be the equilibrium resources, transfer, obtained by a wife of status $f$ under marital regime $X$. The residual output to the type $m$ husband is $\pi_{mf}n^\beta - n\tau_f^X$. Since there is no externality in this society, posing men as residual claimants in the marriage market has no substantive significance.

Given $f$, a type $m$ husband will choose $n$ to solve:

\[
V_{mf}(\tau_f^X) = \max_n \pi_{mf}n - \tau_f^X n
\]

The optimal number of wives, $n_{mf}(\tau_f^X)$, is:

\[n_{mf}(\tau_f^X) = \left(\frac{\beta \pi_{mf}}{\tau_f^X}\right)^\gamma, \quad \gamma = \frac{1}{1 - \beta} > 1\]
\[ V_{mf}(\tau_f^X) = (\beta^{-1} - 1)(\beta \pi_{mf})^\gamma (\tau_f^X)^{1-\gamma} \]  

We will show that for all parameter values, \(0 \leq \ln \pi_{lh} \leq \ln \pi_{hh}\) and \(0 \leq \ln \pi_{hl} \leq \ln \pi_{hh}\), marital equilibria exists. Furthermore, monogamy and polygyny occur in non-overlapping regions of the parameter space. In particular, monogamy exists when \(\pi_{lh}\) and \(\pi_{hl}\) are sufficiently low relative to \(\pi_{hh}\). Otherwise, polygyny prevails. Empirically, we can study the incidence of monogamy in a society as a function of \(\pi_{lh}, \pi_{hl}\), and \(\pi_{hh}\) in that society.

This paper will now consider the different types of marital regimes that may exist in this society.

3 Monogamy A

In any monogamy equilibrium, each married individual will have one spouse. Under Monogamy A, there is positive assortative matching in marriage by social class.

Since each man has one wife, (6) implies:

\[ 1 = \left( \frac{\tau_f}{\beta \pi_{mf}} \right)^{-\gamma} \]  
\[ \tau_h^A = \beta \pi_{hh} \]  
\[ \tau_l^A = \beta \pi_{ll} \leq \tau_h^A \]  

(10) implies that high ability women will not want to mimic low ability women.

For a high status man to not want to marry down implies:

\[ V_{hh}(\tau_h^A) > V_{hl}(\tau_l^A) \]  
\[ \pi_{hh}^{1-\beta} > \pi_{hl} \]  

For a low status man to not want to marry up:

\[ V_{lh}(\tau_l^A) < V_{ll}(\tau_l^A) \]  
\[ \pi_{lh} < \pi_{hh}^\beta \]  

(12) and (14) imply that if mixed status marriages are unproductive relative to high status marriages, monogamy A will exist.

Given a value of \(\beta\), the area in which monogamy A exists is denoted by \(M_A\) in Figure 1. The intuition for positive assortative matching by social status is clear. Since mixed status matches are relatively unproductive (in spite of
a potential for multiple spouses), high status individuals prefer to marry each other. Low status individuals are left to marry each other. Since the sex ratio is one for each social class, there is monogamy.

4 Monogamy $B$

Here, I consider the possibility of monogamy with imperfect negative assortative matching. Because $\alpha < \frac{1}{2}$, high status men can all marry low status women but low status men must marry both high and low status women. Perfect monogamous negative assortative matching, where only mix status marriages exist cannot occur in this society because there are more low status individuals than high status individuals.

In order to satisfy marriage market clearing, the demands for wives must
equal one for all types of marriages:

\[ 1 = \left( \frac{\tau_f}{\beta \pi_{mf}} \right)^{-\gamma} \]  

(15)

\[ \tau_t^B = \beta \pi_{hl} \]  

(16)

\[ \tau_l^B = \beta \pi_{ll} \]  

(17)

\[ \tau_h^B = \beta \pi_{lh} \]  

(18)

\[ \pi_{hl} = \pi_{ll} \]  

(19)

(19) requires that \( \pi_{hl} = \pi_{ll} \) which is due to monogamy and the indifference of low status men between the two types of spouses. But this requirement makes negative assortative matching unlikely for most parameter values.

High status women will not want to mimic low status women:

\[ \tau_h^B \geq \tau_l^B \]  

(20)

\[ \pi_{lh} \geq \pi_{hl} = 1 \]  

(21)

High status men do not want to marry their own type:

\[ V_{hh} \leq V_{hl} \]  

(22)

\[ (\pi_{hh})^\gamma (\pi_{lh})^{1-\gamma} \leq \pi_{hl} = 1 \]  

(23)

The above inequality cannot hold because (20) implies \( \pi_{lh} \geq 1 \) and we also know \( \pi_{hh} > 1 \). So the left hand side of (23) is greater than one. Monogamy \( B \) does not exist.

One way to view the non-existence of monogamy \( B \) is that it has to be an equilibrium choice. But when polygyny is feasible, even imperfect negative assortative matching under monogamy is dominated by polygyny.

5 Monogamy \( C \)

Under monogamy \( C \), both types of men are indifferent between different types of women as wives and there are all types of marriage matches. In this case, the indifference conditions for both types of men imply:

\[ \tau_h^C = \beta \pi_{hh} = \beta \pi_{lh} \]  

(24)

\[ \tau_l^C = \beta \pi_{ll} = \beta \pi_{hl} \]  

(25)

\[ \pi_{hh} = \pi_{lh} \]  

(26)

\[ \pi_{ll} = \pi_{hl} \]  

(27)

So there is only one point in the parameter space with monogamy \( C \). Since, \( \tau_h^C > \tau_l^C \), high status women will not want to mimic low status women.

Considering all the potential monogamous matching patterns, almost all monogamous societies will have positive assortative matching. Negative assortative matching cannot exist under monogamy.
5.1 Digression on exogenous monogamy

When only monogamy is feasible, \( n = \{0, 1\} \). When a type \( m \) man marries a type \( f \) woman, their marital output becomes \( \pi_{mf} \). Following the above arguments, positive assortative matching under exogenous monogamy occurs when:

\[
\pi_{hh} + \pi_{ll} > \pi_{hl} + \pi_{lh}
\]  
(28)

Negative assortative matching under exogenous monogamy occurs when:

\[
\pi_{hh} + \pi_{ll} < \pi_{hl} + \pi_{lh}
\]  
(29)

\( \tau_h \) and \( \tau_l \) are not uniquely determined. These results are standard in the literature.

The conditions for positive assortative matching under equilibrium monogamy, (12) and (14), imply:

\[
\pi_{hh}^{1-\beta} + \pi_{hl}^{\beta} > \pi_{hl}^{1-\beta} + \pi_{lh}^{\beta}
\]  
(30)

Since

\[
\pi_{hh} + \pi_{ll} = \pi_{hh} + 1 > \pi_{hh}^{1-\beta} + \pi_{lh}^{\beta}
\]  
(31)

(30) is significantly stronger than (28).

So (28) is not sufficient for equilibrium monogamy. Nor does equilibrium monogamy with negative assortative matching exists when (29) is satisfied. Thus the claim that the sexual division of labor within a household imply negative assortative matching by wages or education in a monogamous society is premature. Rather, monogamous societies will have positive assortative matching by spousal abilities. Whether or not a monogamous society also practice the sexual division of labor is irrelevant for predicting its marriage matching pattern.

I will now turn to the cases where (12) and (14) fail.

6 Polygyny

Each high status man marry more than one high status woman. Some high status men marry low status women. So high status men are indifferent between the two types of women. Each low status man marry less than one low status woman. So low status men prefer low status wives. This form of polygyny is perhaps the most commonly envisioned, where high status men marry high and low status women.

The indifference of high status men implies:

\[
V_{hh} = V_{hl}
\]

\[
\tau_h^a = \left( \frac{\pi_{hh}}{\pi_{hl}} \right) \tau_l^a
\]  
(32)

(33)
(33) says that high status women will not want to mimic low status women. Let \( \phi \) be the endogenous fraction of high status men that marry high status women. Then market clearing for high status women imply:

\[
\phi \frac{n_{hh}(\tau_h^a)}{n_{hh}(\tau_h^a)} = 1
\]

\[
\phi = \frac{1}{n_{hh}(\tau_h^a)} = \left( \frac{\tau_h^a}{\beta \pi_{hh}} \right) \gamma
\]

The fraction of high status men who marries high status women is increasing in \( \tau_h^a \). The intuition is as follows. As the opportunity cost of high status wives increases, the demand for high status wives per high status husband falls. So more high status husbands are needed to clear the market for high status wives.

Market clearing for low status women imply:

\[
\alpha(1 - \phi)\frac{n_{hl}(\tau_l^a)}{n_{hl}(\tau_l^a)} + (1 - \alpha)\frac{n_{ll}(\tau_l^a)}{n_{ll}(\tau_l^a)} = 1 - \alpha
\]

The market clearing equation for low status women may be written as:

\[
\alpha(1 - \frac{1}{n_l(\tau_l^a)})\frac{n_{hl}(\tau_l^a)}{n_{hl}(\tau_l^a)} = (1 - \alpha)(1 - \frac{n_{ll}(\tau_l^a)}{n_{ll}(\tau_l^a)})
\]

\[
\alpha(1 - \frac{\tau_l^a}{\beta \pi_{hl}})\gamma(\frac{\tau_l^a}{\beta \pi_{hl}})^{-\gamma} = (1 - \alpha)(1 - \frac{\tau_l^a}{\beta \pi_{hl}})^{-\gamma}
\]

Substituting for \( \tau_l^a \),

\[
\alpha(1 - \frac{1}{n_l(\tau_l^a)})\frac{n_{hl}(\tau_l^a)}{n_{hl}(\tau_l^a)} = (1 - \alpha)(1 - \frac{\tau_l^a}{\beta \pi_{hl}})^{-\gamma}
\]

\[
\frac{(\tau_l^a)}{\beta} = \frac{(1 - \alpha) + \alpha \pi_{hl}^\gamma}{1 - \alpha + \alpha \pi_{hl}^\gamma}
\]

\[
\frac{(\tau_l^a)}{\beta} = \frac{n_{hh}(\tau_{hh})}{n_{hl}(\tau_{hl})} \frac{(1 - \alpha) + \alpha \pi_{hl}^\gamma}{(1 - \alpha) + \alpha \pi_{hl}^\gamma}
\]

The demand for \( hh \) marriages is:

\[
n_{hh} = \frac{\alpha + (1 - \alpha)(\frac{n_{hl}}{\pi_{hh}})^{\frac{1}{\gamma}}}{\alpha + (1 - \alpha)\pi_{hl}^\gamma}
\]

Using (41), for \( \phi < 1 \), we need:

\[
\pi_{hl} > \pi_{hh}^{1 - \beta}
\]

which is the same as (12). I.e. if monogamy \( A \) is an equilibrium, polygyny \( a \) is not an equilibrium.

Low status men will prefer not to marry up if:

\[
\pi_{hl}\pi_{hl} < \pi_{hh}
\]
(43) and (42) imply:
\[ \pi_{lh} < \pi_{hh}^{\beta} \]  \hfill (44)

which is (14).

The set of parameter values covered by polygyny a is shown in the bottom right triangle in Figure 1. Since high status men marry high and low status women, it is not surprising that this form of polygyny occurs when \( \pi_{hl} \) is relatively high. It also requires that \( \pi_{lh} \) be relatively low. If high status women are also productive in mix status marriages, they will not be satisfied to just marry high status men.

7 Polygyny b

High status men marry low status women and high status women. They marry all the low status women but only some of the high status women. Low status men marry high status women. Low status men prefer high status wives to low status wives. Polygyny b, if it exists, exhibits imperfect negative assortative matching because some high status men marry high status women.

The indifferece of high status men implies:

\[ V_{hh} = V_{hl} \]  \hfill (45)
\[ \tau_{hl}^b = \left( \frac{\pi_{hl}}{\pi_{hh}} \right)^{\frac{1}{\pi}} \]  \hfill (46)

So high status women will not want to mimic low status women.

Let \( \phi \) be the endogenous fraction of high skill men that marry low skill women. Market clearing for low skill women imply:

\[ \alpha \phi \tilde{n}_{hl}(\tau_{hl}^b) = 1 - \alpha \]
\[ \alpha \phi = \frac{1 - \alpha}{\tilde{n}_{hl}(\tau_{hl}^b)} = (1 - \alpha) \left( \frac{\tau_{hl}^b}{\beta \pi_{hl}} \right)^\gamma \]  \hfill (47)

Market clearing for high skill women imply:

\[ \alpha(1 - \phi)\tilde{n}_{hh}(\tau_{hh}^b) + (1 - \alpha)\tilde{n}_{hl}(\tau_{hl}^b) = \alpha \]  \hfill (48)

The market clearing equation for high skill women may be written as:

\[ \alpha = (\alpha - (1 - \alpha)(\frac{\tau_{hl}^b}{\beta \pi_{hl}})^\gamma)\tilde{n}_{hh}(\tau_{hh}^b) + (1 - \alpha)\tilde{n}_{hl}(\tau_{hl}^b) \]  \hfill (49)
\[ \left( \frac{\tau_{hl}^b}{\beta} \right)^\gamma = \frac{\alpha \pi_{hh} \gamma + (1 - \alpha) \pi_{hl}^\gamma}{\alpha + (1 - \alpha)(\frac{\pi_{hh}}{\pi_{hl}})^{-\frac{1}{\gamma}}} \]  \hfill (50)
Want \( \phi < 1 \):

\[
\begin{align*}
\left( \frac{\tau_{hl}^b(\frac{\pi_{hh}}{\pi_{hl}})^{1/\gamma}}{\beta \pi_{hl}} \right)^{-\gamma} & > \frac{1 - \alpha}{\alpha} \\
\alpha^2 \left( \frac{\pi_{hh}}{\pi_{hl}} \right)^{1/\gamma} & > (1 - \alpha)^2 \left( \frac{\pi_{lh}}{\pi_{hh}} \right)^\gamma
\end{align*}
\]

Because \( \alpha < 1 - \alpha \), a necessary condition is:

\[
\begin{align*}
\left( \frac{\pi_{hh}}{\pi_{hl}} \right)^{1/\gamma} & > \left( \frac{\pi_{lh}}{\pi_{hh}} \right)^\gamma \\
\pi_{hh} & > \pi_{hl} \pi_{hh}
\end{align*}
\]

Since \( \pi_{hh} > \pi_{hl} \) and \( \pi_{hh} > \pi_{lh} \), (54) is always satisfied.

Low status men must prefer high status women to low status women:

\[
\begin{align*}
(\pi_{lh})^\gamma \left( \frac{\pi_{hh}}{\pi_{hl}} \right)^{1 - \gamma} & > (\pi_{lh})^\gamma \left( \frac{\pi_{hl}}{\pi_{hh}} \right)^{1 - \gamma} \\
\pi_{lh} & > \pi_{hh}
\end{align*}
\]

which is the opposite of (43). The substantive content of (56) is shown in Figure 1. Polygyny b, with imperfect negative assortative matching, exists when both types of mixed status matches are relatively productive. Of course this result is unsurprising.

8 Polygyny c

In this equilibrium, there is perfect negative assortative matching. All high status men marry low status women. All low status men marry high status women. When polygyny is feasible, there is the potential for perfect negative assortative matching.

Market clearing for low skill women imply:

\[
\begin{align*}
\alpha \widehat{\pi}_{hl}(\tau_c^l) & = 1 - \alpha \\
\tau_c^l & = \beta \pi_{hl} \left( \frac{\alpha}{1 - \alpha} \right)^{1 - \beta}
\end{align*}
\]

Market clearing for high skill women imply:

\[
\begin{align*}
(1 - \alpha) \widehat{\pi}_{lh}(\tau_c^h) & = \alpha \\
\tau_c^h & = \beta \pi_{lh} \left( \frac{1 - \alpha}{\alpha} \right)^{1 - \beta}
\end{align*}
\]

For high status women not to want to mimic low status women, 

\[
\pi_{lh} \left( \frac{1 - \alpha}{\alpha} \right)^{1 - \beta} > \pi_{hl} \left( \frac{\alpha}{1 - \alpha} \right)^{1 - \beta}
\]
High status men must prefer low status women to high status women:

\[
(\pi_{hh})^\gamma (\pi_{lh}(\frac{1-\alpha}{\alpha})^{1-\beta})^{1-\gamma} < (\pi_{hl})^\gamma (\pi_{hl}(\frac{\alpha}{1-\alpha})^{1-\beta})^{1-\gamma}
\]  

(62)

(61) and (62) lead to a contradiction. Polygyny (c) does not exist. Thus there is no polygynous equilibrium with perfect negative assortative matching.

9 Polygyny \(d\)

All high status men marry high status women. Low status men marry low and high status women.

Indifference of low status men implies:

\[
V_{lh} = V_{ll}
\]

\[
\tau^d_h = (\frac{\pi_{lh}}{\pi_{ll}})^\frac{1}{\gamma} \tau^d_l
\]  

(64)

High status women will not want to mimic low status women.

Let \(\phi\) be the endogenous fraction of low status men that marry low status women. Market clearing for low status women imply:

\[
\phi \tilde{n}_h(\tau^d_l) = 1
\]

\[
\phi = \frac{1}{\tilde{n}_h(\tau^d_l)} = (\frac{\tau^d_l}{\beta \pi_{ll}})^\gamma
\]  

(65)

The fraction of low status men who marries low status women is increasing in \(\tau^d_l\).

Market clearing for high status women imply:

\[
\alpha \tilde{n}_{hh}(\tau^d_h) + (1-\alpha)(1-\phi)\tilde{n}_{lh}(\tau^d_h) = \alpha
\]  

(66)

which may be rewritten as:

\[
\alpha = \alpha(\frac{\tau^d_h}{\beta \pi_{hh}})^{-\gamma} + (1-\alpha)(1-\frac{\tau^d_l}{\beta \pi_{ll}})^{-\gamma} (\frac{\tau^d_h}{\beta \pi_{lh}})^{-\gamma}
\]  

(67)

\[
(\frac{\tau^d_h}{\beta})^{-\gamma} = \frac{\alpha \pi_{hh} + (1-\alpha)\pi_{lh}^{\frac{1}{\gamma}}}{\alpha + (1-\alpha)(\frac{1}{\pi_{lh}})^{\frac{1}{\gamma}}}
\]  

(68)

We need \(\phi < 1\), low skill men to prefer more than one low status wife each:

\[
(\frac{\tau^d_l}{\beta \pi_{ll}})^{-\gamma} > 1
\]  

(69)

\[
\frac{\alpha \pi_{lh}^{\frac{1}{\gamma}} + (1-\alpha)}{\alpha(\frac{\tau^d_h}{\pi_{hh}})^{\gamma} + (1-\alpha)} > 1
\]  

(70)
So we need
\begin{align}
\pi_{lh}^\gamma & > \pi_{hh}^\gamma \tag{71} \\
\pi_{lh} & > \pi_{hh}^\beta \tag{72}
\end{align}

which is the opposite of (14).

We also want high status men to prefer high status wives rather than low status wives:
\[ \pi_{hh} > \pi_{lh}\pi_{hl} \tag{73} \]

which is the same as (43).

Figure 1 shows where polygyny \( d \) exists.

## 10 Polygyny \( e \)

All high status men marry low status women. Low status men marry low and high status women. The indifference of low status men implies:
\begin{align}
V_{lh} &= V_{ll} \tag{74} \\
\tau^e_h &= \left( \frac{\pi_{lh}}{\pi_{ll}} \right)^\frac{1}{\beta} \tau^e_l \tag{75}
\end{align}

High status women will not want to mimic low status women.

Let \( \phi \) be the fraction of low status men that marry all the high status women. Then:
\[ (1 - \alpha)\phi n_{lh}(\tau^e_h) = \alpha \]
\[ \phi = \frac{\alpha}{1 - \alpha} \left( \frac{\tau^e_h}{\beta \pi_{lh}} \right)^\gamma \tag{77} \]

Market clearing for low status women imply:
\begin{align}
(1 - \alpha)(1 - \phi)n_{ll}(\tau^e_l) + \alpha n_{hl}(\tau^e_l) &= 1 - \alpha \tag{78} \\
(1 - \alpha) + \alpha \pi_{hl}^\gamma &= (\frac{\tau^e_l}{\beta})^\gamma \tag{79}
\end{align}

We want \( \phi < 1 \):
\[ \frac{(1 - \alpha) + \alpha \pi_{hl}^\gamma}{(1 - \alpha) + \frac{(1 - \alpha)^2 - \frac{\alpha^2}{2}}{\pi_{lh}}} < 1 \]
\[ \pi_{lh} \pi_{hl}^\gamma < \frac{(1 - \alpha)^2}{\alpha^2} \tag{81} \]

So we need:
\[ \pi_{lh}^\frac{1}{\gamma} \pi_{hl}^\gamma < \frac{(1 - \alpha)^2}{\alpha^2} \tag{81} \]

Since the right hand side of (81) is unbounded, (81) can always be satisfied for a sufficiently small \( \alpha \).
We want high status men to prefer low status wives:

\[(\pi_{hh})^{\gamma}(\tau_h^g)^{1-\gamma} < (\pi_{hl})^{\gamma}(\tau_l^g)^{1-\gamma}\]  

\[\pi_{hh} < \pi_{lh}\pi_{hl}\]  

(83) is the same as (56).

Whenever polygyny \(e\) exists, polygyny \(b\) also exists. Polygyny \(b\) only requires (56). Thus when (56) and (81) hold, there are multiple equilibria. Polygyny \(e\) need not exist. If \((1-\alpha)^2\alpha^{-2} < \pi_{hh}^g\), polygyny \(e\) will not exist.

11 Polygyny \(g\)

High status men are indifferent between high status and low status wives. Low status men are also indifferent between high status and low status wives.

The indifference of high status men between the two types of wives implies:

\[V_{hh} = V_{hl}\]  

\[\tau_h^g = \left(\frac{\pi_{hh}}{\pi_{hl}}\right)^{\frac{1}{\gamma}}\tau_l^g\]  

High status women will not want to mimic low status women.

The indifference of low status men between the two types of wives implies:

\[V_{lh} = V_{ll}\]  

\[\tau_l^g = \left(\frac{\pi_{lh}}{\pi_{ll}}\right)^{\frac{1}{\gamma}}\tau_l^g\]

(85) and (87) imply:

\[\pi_{hh} = \pi_{lh}\pi_{hl}\]

Let \(\phi_h\) be the fraction of high status men with high status wives. Let \(\phi_l\) be the fraction of low status men with high status wives. Then market clearing for high status wives imply:

\[\alpha\phi_h n_{hh}(\pi_h^g) + (1-\alpha)\phi_l n_{lh}(\pi_l^g) = \alpha\]  

\[\alpha\phi_h \pi_{hh}^\gamma + (1-\alpha)\phi_l \pi_{lh}^\gamma = \left(\frac{\pi_h^g}{\beta}\right)^\gamma\alpha\]

Market clearing for low status wives imply:

\[\alpha(1-\phi_h)\pi_{hl}^\gamma + (1-\alpha)(1-\phi_l) = \left(\frac{\pi_l^g}{\beta}\right)^\gamma(1-\alpha)\]
(90) and (91) imply:

\[
\alpha \pi_{hl}^\gamma + (1 - \alpha) = \left(\frac{\pi_{lh}^g}{\beta \pi_{lh}}\right)^\gamma \alpha + \left(\frac{\pi_{lh}^g}{\beta}\right)^\gamma (1 - \alpha) \quad (92)
\]

\[
\left(\frac{\pi_{lh}^g}{\beta}\right)^\gamma = \frac{\alpha \pi_{hl}^\gamma + (1 - \alpha)}{\alpha \pi_{lh}^\gamma + (1 - \alpha)} \quad (93)
\]

\[
\left(\frac{\pi_{lh}^g}{\beta}\right)^\gamma = \frac{\alpha \pi_{lh}^\gamma + (1 - \alpha)}{\alpha \pi_{lh}^\gamma + (1 - \alpha)} \quad (94)
\]

Again using (90) and (91):

\[
\frac{\alpha \pi_{hl}^\gamma \phi_h + (1 - \alpha) \phi_l}{\alpha \pi_{hl}^\gamma + (1 - \alpha)} = \frac{\alpha (\pi_{lh}^g)^1}{\alpha (\pi_{lh}^g)^\gamma + (1 - \alpha)} < 1 \quad (95)
\]

\(\phi_l\) and \(\phi_h\) are not unique. The set of equilibria is defined by the locus (95) subject to the restriction that both \(\phi_l\) and \(\phi_h\) must be positive fractions. For all admissible values of \(\pi_{hl}\) and \(\pi_{hl}\), equilibria exist.\(^{12}\)

### 12 Discussion

For all parameter values, \(0 \leq \pi_{lh} \leq \pi_{hh}\) and \(0 \leq \pi_{hl} \leq \pi_{hh}\), marital equilibria exist. Figure 1 shows where the different equilibria occur. Multiple polygamous equilibria exist, polygyny \(b\) and \(e\), for a subset of the parameter space.

Except for one point in the parameter space, all monogamy equilibria exhibit positive assortative matching in marriage by social class, monogamy \(A\). Monogamy \(A\) exists when \(\pi_{lh}\) and \(\pi_{hl}\) are sufficiently low relative to \(\pi_{hh}\). This result was anticipated by Becker’s classic discussion of the marriage market. Unlike Becker, when marital regimes are endogenous, except for one point, polygyny occurs when monogamy \(A\) fails to be an equilibrium.

That monogamy and polygyny occur in non-overlapping regions of the parameter space is useful for empirical testing of the model. In particular, monogamy exists when \(\pi_{lh}\) and \(\pi_{hl}\) are sufficiently low relative to \(\pi_{hh}\). Otherwise, polygyny prevails.

There are different types of marriage matching patterns under polygyny. Interestingly, although perfect negative assortative matching under polygyny is feasible, it does not exists in equilibrium. A conclusion of this paper is that perfect negative assortative matching does not exists. Such a conclusion is surprising given Becker’s results under exogenous monogamy.

#### 12.1 Why are modern industrial societies monogamous?

This paper is too simple and abstract to explain why modern industrial societies are monogamous. Specific mechanisms, such as that explored in Gould, et. al.,

\(^{12}\)The equilibrium \(\phi_h = \phi_l = \frac{\alpha (\pi_{lh}^g)^1}{\alpha (\pi_{lh}^g)^\gamma + (1 - \alpha)}\) exists for all admissible values of \(\pi_{lh}\) and \(\pi_{hl}\).
are needed for such a task. Rather, this paper clarifies what these mechanisms must satisfy to provide an explanation.

The paper does provide a link between Becker’s suggestion on why modern society are monogamous in a model where only men are heterogenous, and models where women are also heterogenous.

Consider a society in which there are high and low ability men, indexed by their wage, and homogenous women. Further assume that the child quality production function is increasing and concave in goods and paternal time investment. High wage fathers, who have a high cost of time, should economize on paternal time (investing in the quality of children). Instead, they should substitute toward a larger quantity of lower quality children (polygyny).

As discussed earlier, in unpublished work with Jeanne Lafortune, we found that the society can be monogamous if there is strong increasing returns to paternal investment and or the degree of increasing returns is strongly increasing in father’s ability. In this case, high wage men may want to economize on the quantity of children and therefore monogamy obtains.

When women are also heterogenous, there is an additional effect as shown in this paper. When high ability wives produces high quality children but low ability wives do not, it provides an alternative way obtaining child quality. Instead of requiring increasing returns to paternal time, the high ability male can obtain high quality children by attracting a high ability wife. So if low ability wives are poor substitutes for high ability wives in producing child quality and child quality is sufficiently valued, high ability men will marry high ability women and be monogamous.

What remains to be answered is why high ability wives are so much productive in producing higher quality children now than in less modern societies. I speculate that the following forces are important: the economic independence of high ability women, ease of divorce and modern child custody laws that favor mothers.

I do not want to argue that the above sketch is the correct explanation for monogamy in modern societies. The objective of the sketch is to demonstrate how the framework in this paper can be used to think about specific behavioral mechanisms.

References


