Quality Signals, Competition and Consumer Fraud*

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Abstract

This paper considers a two-sided private information model. We discuss the signaling role of consumers' private but imperfect information together with the effect of competition on the equilibrium level of honesty and incidence of fraud. We assume that two exogenously given qualities, which are represented by sellers types, are offered in the market. Prices are fixed. Low quality sellers choose to be either honest (by charging the lower market price) or dishonest (by charging the higher price). We demonstrate that in equilibrium, the amount of fraud might increase when the precision of the buyer's private information increases. Furthermore, we show that the level of dishonesty is non-decreasing in the level of market competition in equilibrium.

Keywords: Quality Uncetainty; Price Signalling; Imperfect Quality Signals; Adverse Selection; Honesty; Competition

JEL Classification: C72; D42; D82; G14; L15

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1 Introduction

The market is the place set apart where men may deceive one another.

Anacharsis, 600 B.C.

Consumer fraud is a common occurance. The National Institute of Justice sponsored a November 1991 survey of a random sample of 1,246 consumers aged 18 and subsequently[1]. This survey found that 15% of participants had been the victim of a successful personal fraud[2] in the 12 months prior to the interview and 13% of them reported suffering direct monetary injury. The Federal Trade Commission (FTC) commissioned a 2003 survey of 2,500 randomly chosen adults about their consumer experiences during the previous year by targeting specific types of fraud[3]. The results suggest that 25 million of adult Americans (around 11.2% of adult population) were victims of one or more of the fraud actions covered by the survey[4]. Consumer fraud is also common outside the United States[5].

Our theoretical development is motivated by this evidence. The major objective of this paper is to determine the role played by the accuracy of consumers' private information and the effect of competition on the equilibrium levels of honesty and incidence of fraud.

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[1]The results of this survey have been published in Titus, Heinzelmann and Boyle (1995).

[2]The survey asked about "local" fraud problems, such as problems with automobile or appliance repairs and with home repairs and improvements.

[3]The most frequently reported types of consumer fraud were: telephone slamming, advance fee loan scams, being billed for some membership without having agreed for it, credit card insurance and credit repair.

[4]The American Association of Retired Persons (AARP) conducted a survey in December of 1998 involving interviews with 1,504 adults over the age of 18. Three quarters of those in the study reported that they had at least one bad experience when buying a product or service in the year preceding the interview. Further, 17% answered that they were the subject of a major consumer fraud.

[5]On May 9th, 2006, Spain's Public Prosecutor accused Afinsa, Bienes Tangibles S.A., the world's largest stamp dealer, of operating the biggest scam in Spanish history. For years, the company invited investors in many Spanish cities and towns to buy antique stamps that were completely overvalued if not forged and then paid them interest that did not come from appreciation in stamp value as promised but rather from money chipped in by new customers. Almost 1% of the Spanish population had entrusted their savings to the stamp company.

To address these questions, we introduce a simple price-quality signaling model of an experience good with private information on both sides of the market. A seller has a good whose quality is exogenously either high or low. We assume that quality realizations are distributed independently and identically across time. This description typically fits a market for an experience good whose quality is subject to some stochastic process, being examples, products which are generated by R&D, second-hand objects, crops, or even a fish. The seller knows the quality of his[6] good and he demands one of the two exogenous prices p_L and $p_H > p_L$ for his good. A consumer observes the price demanded and a noisy binary signal of quality and chooses whether to purchase the good. We assume that the consumer is not willing to pay the high price for a low quality product.

We analyze a static monopolistic setting first and introduce competition in a dynamic setting subsequently. A key feature of the model is that competition exists only on the sellers' side of the market. Furthermore, trade in the current period guarantees any seller the visit of a customer in the following period. If trade does not take place, the seller's commercial relations with the buyer cease and a random matching technology is used to describe the meeting between the customer and any of the unemployed sellers in the following period. We do not consider reputation concerns and leave their introduction for future research.

In this context, the informational asymmetries create obvious incentives for opportunistic behavior by the sellers: a seller with a low quality good could swindle the buyer by charging the high price.

In the first part of the paper, we are concern about the effect of the accuracy of the buyer's private information on consumer fraud. It seems intuitive to conjecture that an increase in the accuracy of the buyer's private information reduces the probability the buyer is swindled. If the buyer becomes better informed about the quality of the object for sale, the seller will be less successful in misrepresenting it and he will be able to deceive the buyer less frequently. This lower probability of trade should restrain the seller from overcharging and induce him to be more honest. As we will show in this paper, this argument is incomplete. In equilibrium, the amount of fraud

[6]For ease of exposition, we refer to a seller as 'he' and to a consumer as 'she'.

might increase when the precision of the buyer's private information increases. This result has an intuitive explanation. As the signal's precision increases, a buyer who receives a positive signal becomes more convinced of the product's good quality while the opposite is true when the buyer receives a negative signal. When, in equilibrium, only a buyer with a positive signal is willing to pay the high price, an increase in the signal's quality increases the willingness to pay of such buyer, and ultimately induces the low quality seller to attempt a fraud more often. In this case, more information is beneficial from the seller's point of view.

The second part of the paper focuses on the case when there is more than one seller, and it investigates the effect of competition on consumer fraud. In this model, one would expect competition to discipline sellers. With competition, demanding a high price results in a larger probability of being unemployed in the next period because it reduces the chances of carrying out a successful trade. This, in fact, increases the relative long-run cost of cheating. Notice that because of the noisy signals, an honest seller with a high quality good may be accidentally dismissed. However, the probability of dismissal is higher for a dishonest seller than for an honest one because the signal is imperfectly correlated with the quality of the good. Our characterization illustrates how three equilibria coexist for some parameter range (while there was a unique equilibrium under monopoly). When there are multiple equilibria, the probability of fraud is constant in the market competition level for two of the equilibria and grows in the market competition in the third equilibrium. This result is explained by noting that the probability of remaining unemployed in the subsequent periods depends negatively on the rate of turnover among sellers, and the latter increases with the probability that each seller attempts a fraud. Thus, the gains from being employed relative to unemployed, and ultimately, the long-run cost of cheating, are decreasing in the probability that each seller attempts a fraud. Consequently, an increase in competition can coexist with an increase in the amount of fraud when their opposite effects on the relative long-run cost of cheating balance each other.

In brief, our findings are three-fold. First, equilibria involving fraud exist for all possible combinations of competition and signal precision levels. Second, the amount of fraud might increase when the precision of the buyer's private information increases. Third, the level of dishonesty is non-decreasing in the level of market competition in equilibrium.

The paper is organized as follows. Section 2 provides a literature review. The model is formalized in section 3 and some preliminaries are introduced in section 4. Section 5 carries out the equilibrium analysis and presents the central results of the paper. Section 6 concludes with a discussion of directions for future research. Proofs are contained in the Appendix.

2 Related Literature

The market mechanism under asymmetric information caused by quality uncertainty has constituted a main focus of research in the economics of information. The notion of experience goods was introduced by Nelson (1970, 1974). The quality of an experience good cannot be directly verified before purchase (ex ante), but only after purchase (ex post) through the use of the product. The quality distribution for the good in the market is assumed to be known, however.

When deciding whether to buy a good of dubious quality, consumers often rely on any additional information that can be used to evaluate quality. Numerous experimental studies show that consumers infer a high quality from a high price. This fact contributed to the development of the industrial organization literature on price as a signal of quality. The existing work can be embodied into two main areas of research: a first area is concerned with the moral hazard aspects of the choice of quality, while the second focuses on the classical adverse selection problem, pioneered by Akerlof (1970). The latter line of research is the one pursued in this paper. In these models, quality is not treated as a choice variable but, instead, is exogenously given. The vast majority of the papers that belong to this body of literature[7] assume the potential customers to be either perfectly informed

[7] Refer to Milgrom and Robers (1986), Bagwell and Riordan (1991), Overgaard (1993), Ellingsen (1997), Bester and Ritzberger (1999), Hertzerdof and Overgaard (2001), Daughety and Reinganum (2004).

or completely uninformed about the quality of the good on sale. Several papers (Wolinsky (1983), Judd and Riordan (1994), Voorneveld and Weibull (2004)) relax this assumption. Our modelling approach is in the spirit of Voorneveld and Weibull (work): we specify that each consumer observes a private and imperfect binary signal without incurring any cost. As a result, the model is categorized as a two-sided asymmetric information model. The central question addressed by the price-quality signaling models consists on determining whether in such settings, prices alone are capable of conveying information on quality in equilibrium. Therefore, they restrict attention to pure strategy sequential equilibria and investigate the existence of fully separating equilibrium outcomes that survive selection criteria. Thus, in such equilibrium outcomes, honest reporting prevails in the market. In contrast, we focus on pooling and mixed strategies, which may exhibit various degrees of fraud. Furthermore, we perform comparative statics in terms of different parameters of interest in order to determine whether more or less information is revealed in equilibrium, and calculate its impact on the level of fraud in the market.

There is some previous work on the issue of honesty and competition in the area concerned with the moral hazard aspects of the choice of quality. Cooper and Ross (1984) consider honesty in a static quality provision model under perfect competition. In such a setting, a rational expectations equilibrium may not exist and when it does, the informational content of prices depends on the shape of the average cost curves and the exogenous proportion of informed buyers. Bandyopadhyay (2004) examines a dynamic environment in which investment in quality is costly. A unique fixed price is assumed for both goods so that selling the low quality object at this price is referred as cheating. Dishonest sellers who are matched die at the end of the period with certainty while for honest or unmatched sellers, there is a strictly positive probability of survival. In this context, the author derives conditions under which multiple steady states emerge involving different levels of quality (and thus, honesty) and market thickness (the expected number of customers who accept to trade per seller). The sufficient condition for multiplicity ensures that more market thickness can never induce less honesty.

An alternative strand of related literature that focuses on the analysis of fraud includes work on credence goods[8]. With credence goods (Darby and Karni (1973)), consumers cannot judge actual quality either before or after purchase. Two potential problems may emerge under theses circumstances: provision of a wrong quality or inefficient treatment (under/overtreatment) and charging for a higher quality or more expensive treatment than provided (overcharging). Under a liability assumption, the result (whether the client's problem has been fixed or not) is assumed to be verifiable and consumers are protected from undertreatment. Thus, if the cost of treatment is increasing in quality, overcharging is strictly more profitable than overtreatment. Consequently, an expert who faces a consumer with a minor problem explicitly chooses to be honest by reporting the minor problem and charging for the inexpensive treatment or to be dishonest by reporting the minor problem as serious and charging the expensive treatment but providing the inexpensive one. Pitchik and Schotter (1987 and 1993) identify equilibria in which sellers mislead some of their customers in a regulated market with fixed prices. In their more recent paper, multiple sellers compete for consumers who have the option to search for second opinions. In this static setting, the cost of search could be interpreted as a natural measure of competition, the market being considered as more competitive the lower the cost of consumer search. The authors find that a reduction in the cost of search leads experts to be more honest in equilibrium. Wolinsky (1993) shows that cheating in a competitive and flexible price setting can be eliminated in a static framework when customers search for second opinions or in a stationary dynamic framework in which customers search for second opinions and experts have reputation concerns. For sufficiently low search costs, equilibria give rise to specialization by the experts in the static framework and competition drives the prices of both services to their respective marginal costs. Fong (2005) finds that identifiable customer heterogeneities play a crucial role in explaining expert cheating. He shows that the no cheating result extends to a static flexible price monopolistic setting where customers do not have the option to search for second opinions, nor experts have concerns for reputations and cheating is costless. Under the assumption of identifiable customer characteristics, experts replace

[8]Refer to Dulleck and Kerschbamer (2006) for a unifying framework that can reproduce the majority of results that have been achieved in the literature.

price discrimination by cheating their customers selectively. Specially, those consumers with higher valuations for treatment or those whose problems are more costly to fix, become victims of fraud.

3 The Model

Consider a market for a good (commodity or service) for which quality is the only characteristic relevant to a buying decision. We assume quality is an experience attribute: potential customers are unable to objectively verify the quality of the good before purchase.[9] Time is discrete and the horizon is infinite. We assume that there is only one buyer and $n \geq 1$ sellers in the market. As a result, n captures the level of competition in the market. All agents are infinitely lived and risk neutral. Each seller produces one unit of the good for sale and the potential customer is willing to purchase at most one unit.

We simplify our analysis by assuming that two exogenously given qualities are offered in the market. A good can be of either low quality or high quality. The seller and consumer differ from each other in their valuation for different quality items. The consumer values quality at $0 < v_L < v_H$ respectively. The seller's valuation or reservation price for both types of items is normalized to zero. As a result, there are positive potential gains from trade in both cases.

Sellers are ex ante identical. However, each time that a seller produces a good, it is high quality with probability $\pi \in (0, 1)$ and low quality with probability $1 - \pi$. This probability distribution is common knowledge, independent and identical across time. The unit production costs are also common knowledge and normalized to zero.

The seller knows his actual, realized quality, but his potential customer does not and there is no credible direct way by which the seller can provide this information before the customer makes her purchase decision. However, the seller cannot completely disguise the true quality of his product. Assume that prior to purchase, the consumer obtains without cost a private binary signal, which

[9] For experience goods, quality must be verifiable at least after consumption. Thus, beliefs are given by a probability distribution over quality and are known ex ante or at least, can be deduced from own experience after consumption. (Nelson (1970), Nelson (1974)).

conveys a certain amount of information about product quality. Let the signal structure be common knowledge and be given by:

	Low Quality	High Quality
Low Signal ($s = L$)	δ	$1 - \delta$
High Signal ($s = H$)	$1 - \delta$	δ

where $\frac{1}{2} < \delta \leq 1$.

The number δ is interpreted as the precision of the consumer's signal. In the limit, when it is equal to one, quality can be deduced with certainty by pure inspection before consumption. This is the case of symmetric information. In the other extreme case, if δ were equal to one half, the signal would become totally uninformative since it would be uncorrelated with the quality of the good, corresponding to the case of one-sided asymmetric information. In the intermediate cases in which the signal is imperfectly correlated with the true quality of the product, this model is categorized as a two-sided asymmetric information model.

We study the case in which price is the only signaling variable available to the seller. The seller sets a take-it-or-leave-it price $p \in \{p_L, p_H\}$ for his unit, knowing its quality. Prices are exogenously fixed[10] and satisfy $0 < p_L < v_L < p_H < v_H$, so that under perfect information, the buyer is not willing to purchase a low quality item at a high price. Otherwise, trade is desirable under perfect information. From now on, selling a low quality object at a high price will be referred to as "cheating".

The consumer is Bayesian rational; she has beliefs over the sellers type and she uses all the available information to update her beliefs according to Bayes rule. The buyer is an expected utility maximizer and she bases her trading decision on the expected gains from trade. In each period, the buyer's strategy is simply whether to accept or reject the offer proposed by the seller.

[10]If prices are endogenized, then a plethora of equilibria emerge in a one period monopoly setting and among the refinements suggested in the literature, only criteria D1, which is equivalent Universal Divinity and Never a Weak Best Response in this setting, has any power in pruning the set of outcomes, and its power is limited.

If the offer proposed by the seller is accepted by the buyer, the buyer consumes the good, enjoys its true valuation and the seller and the buyer continue to be matched in the next period. The seller's and buyer's respective (ex post) per period utilities if the offer at price p is accepted in a given period, are thus given by $u_s = p$ and $u_b = v - p$, where $v \in \{v_L, v_H\}$. If the offer is rejected, the buyer does not consume that period. Both agents' utilities are normalized to zero in this case. At the beginning of next period, the buyer meets a seller out of the pool of n unmatched sellers randomly. The assumption regarding the probability associated with a continuation match is based on the fact that past sales are observable. This assumption makes it clear that the model pertains to markets in which stores which are successful in selling their merchandize today, are more likely to receive the visit of a customer tomorrow. For simplicity, the probability of facing a continuation match following trade is normalized to one. A generalization of this assumption does not alter the substance of the conclusions that follow. Figure 1 provides an extensive-form representation of a single period game.

Sellers maximize the expected present discounted value of lifetime earnings. Thus, each period, the matched seller faces the trade-off between charging the low price and obtaining a low one period gain while assuring a match next period (which may lead to future sales), and charging the high price which may yield a high period gain and a continuation match, each with a probability strictly lower than one.

3.1 Equilibrium

We confine attention to Symmetric Stationary Perfect Bayesian Equilibrium (SSPBE). In symmetric equilibria all the agents of the same type play the same strategy. Stationary equilibria are a more restricted class of equilibria where every period, the agents choose a stationary or time invariant strategy upon to move. Let ϕ_H^* and ϕ_L^* denote the probabilities that each type of seller charges the high price in equilibrium. Let $b_H^*(p)$ and $b_L^*(p)$ denote the probabilities that each type of buyer accepts the offer p proposed by the seller in equilibrium.

Definition 1. *A Symmetric Stationary Perfect Bayesian Equilibrium (SSPBE) consists of*

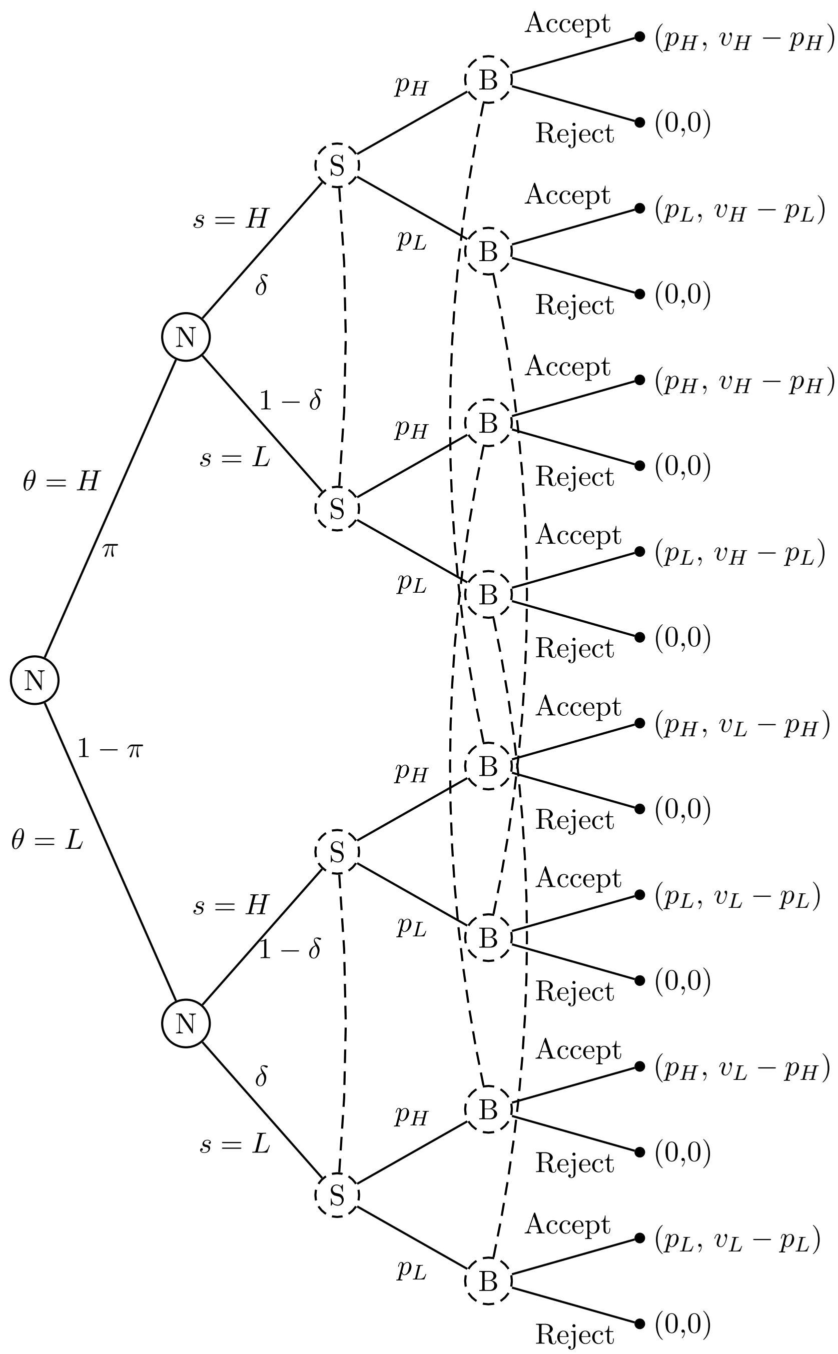

Figure 1: The Extensive-Form Representation of the One Period Signaling Game

beliefs and strategies (μ^*, ϕ^*, b^*) *satisfying the following requirements:*

(A) *Given the players' beliefs, their strategies are sequentially rational. At each information set, the action taken by the player with the move must be optimal given the player's belief at the information set and the other players' subsequent strategies:*

(1) *The seller's strategy* ϕ^* *is a best reply to the buyer's strategy:*

$$\forall\theta, \quad \phi^*_\theta \in \arg\max_\phi V_\theta$$

where $V_\theta \equiv \max\{V_\theta(p_H), V_\theta(p_L)\}$ *denotes the expected discounted lifetime profits for a type* $\theta \in \{L, H\}$ *seller who is matched.*

(2) *The buyer's strategy* b^* *is optimal given* p*,* s *and her associated beliefs:*

$$\forall p, s, \quad b^*_s(p) \in \arg\max_b Eu(p, s, \mu_b(\theta|p, s), b)$$

where $Eu(p, s, \mu_b(\theta|p, s), b) \equiv \sum_{\theta\in\Theta} \mu_b(\theta|p, s)b(v_\theta - p)$ *denotes the buyer's expected utility.*

(B) *At each information set, the player with the move has a belief about which node in the information set has been reached by the play of the game. At information sets on the equilibrium path, beliefs are determined by Bayes' rule and the players' equilibrium strategies:*

(3) *At each seller's information set* $\{H, L\}$*, the system of seller's beliefs satisfies:*

$$\mu^*_s(s|\theta) = \begin{cases} \delta & \text{if } s = \theta \\ 1-\delta & \text{if } s \neq \theta \end{cases}$$

(4) *At each buyer's information set* $\{(H, p_H), (H, p_L), (L, p_H), (L, p_L)\}$*, the system of consumer beliefs is Bayes-consistent, that is,*

$$\mu^*_b(\theta|p_H, s) = \begin{cases} \frac{Pr(\theta)Pr(s|\theta)\phi^*_\theta}{\sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')\phi^*_{\theta'}} & \text{if } \sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')\phi^*_{\theta'} > 0 \\ Arbitrary & \text{if } \sum_{\theta'\in\Theta} Pr(\theta')Pr(s|\theta')\phi^*_{\theta'} = 0 \end{cases}$$

$$\mu_b^*(\theta|p_L, s) = \begin{cases} \frac{Pr(\theta)Pr(s|\theta)(1-\phi_\theta^*)}{\sum_{\theta' \in \Theta} Pr(\theta')Pr(s|\theta')(1-\phi_{\theta'}^*)} & \text{if } \sum_{\theta' \in \Theta} Pr(\theta')Pr(s|\theta')(1-\phi_{\theta'}^*) > 0 \\ Arbitrary & \text{if } \sum_{\theta' \in \Theta} Pr(\theta')Pr(s|\theta')(1-\phi_{\theta'}^*) = 0 \end{cases}$$

4 Preliminaries

4.1 Symmetric Information

The classical case of symmetric information, that is, when the signal is perfect informative, corresponds to the boundary case $\delta = 1$ in the present model. Due to the absence of information asymmetries, a unique separating equilibrium exists. The potential customer's optimal decision involves buying the good if and only if the price posted by the seller does not exceed her willingness to pay. That is, the buyer's optimal strategy is to accept trade at a high price offer if and only if she obtains a high signal realization and accept trade at a low price for every possible signal realization. As a result, the seller's unique best reply to the buyer's strategy is to be honest by setting the low price if he has a low quality good and vice versa. Trade occurs with certainty in equilibrium so that there is no turnover and the expected potential gains from trade are fully realized:

$$\bar{W}^* = \pi v_H + (1-\pi)v_L \tag{1}$$

4.2 The Buyer's Decision Problem

In our setting, competition is restricted to occur only on one side of the market (the sellers' side) since the buyer is guaranteed a match in the following period independently of her current trading decision. Thus, the buyer's decision problem is static due to the sellers' stationary strategies. Let $\bar{v}$ denote the buyer's ex ante expected valuation for a given object: $\bar{v} = \pi v_H + (1-\pi)v_L$. As noted above, once the potential customer observes the price-signal pair (p, s), she updates her beliefs as to which type of seller she faces. The potential customer's optimal decision is to purchase the good if and only if the price posted by the seller does not exceed the ex post expected valuation of the good, which is given by $v_L + \mu_b(H|p, s, \phi)(v_H - v_L)$. As a result, $\forall s \in \{L, H\}$, the buyer's optimal trading decision rule takes the form:

$$b_s^*(p_H) = \begin{cases} 1 & \text{if } \mu_b(H|p_H, s) > A \\ [0,1] & \text{if } \mu_b(H|p_H, s) = A \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and

$$b_s^*(p_L) = 1, \tag{3}$$

where $A \equiv \frac{p_H - v_L}{v_H - v_L} \in (0, 1)$ could be interpreted as a proxy for the extent of fraud (in relative terms) existing in the market, provided that the sellers engage in fraudulent actions. The closer the value of A is to 0, the less severe is the fraud committed by sellers. This is because the price charged by the seller with the low quality good is only slightly above the consumer's true valuation for this good. Likewise, the closer the value of A is to 1, the more severe is the seller's unethical behavior.

4.3 The Seller's Decision Problem

Let V be the expected discounted lifetime profits of a matched seller who behaves optimally before getting to know his type, and V_u the expected discounted lifetime profits of an unmatched seller. Let $\beta \in (0, 1)$ denote the discount factor, which is common across agents. In a stationary environment, the indirect utility of a matched type θ seller of charging the low price is described by Bellman's equation from dynamic programming: $V_\theta(p_L) = p_L + \beta V$. The indirect utilities that each type of matched seller obtains by charging the high price are given respectively by:

$$V_H(p_H) = [\delta b_H^*(p_H) + (1-\delta) b_L^*(p_H)](p_H + \beta V) + [1 - \delta b_H^*(p_H) - (1-\delta) b_L^*(p_H)] \left(0 + \beta \frac{1}{n} V + \beta \left(1 - \frac{1}{n}\right) V_u\right) \tag{4}$$

and

$$V_L(p_H) = [(1-\delta) b_H^*(p_H) + \delta b_L^*(p_H)](p_H + \beta V) + [1 - (1-\delta) b_H^*(p_H) - \delta b_L^*(p_H)] \left(0 + \beta \frac{1}{n} V + \beta \left(1 - \frac{1}{n}\right) V_u\right). \tag{5}$$

The above equations for the high and low quality sellers can be interpreted very easily. For example, at the beginning of a period, a seller with a high quality product expects to confront a customer whose signal realization is high with probability δ and a customer whose signal realization is low with probability $1-\delta$. Thus, $\delta b_H^*(p_H) + (1-\delta) b_L^*(p_H)$ is the probability with which he expects his high price offer to be accepted by the consumer. If the high price offer is accepted, the seller makes a high price profit this period and faces the continuation payoff for the matched seller. If the high price offer is rejected, he makes a zero profit this period and with probability $\frac{1}{n}$, he is rematched to the buyer next period, obtaining the continuation payoff of the matched seller, while with probability $1-\frac{1}{n}$ he does not meet the buyer and obtains the continuation payoff of the unmatched seller. In any period, a matched type θ seller chooses the price strategy that maximizes $V_\theta \equiv \max\{V_\theta(p_H), V_\theta(p_L)\}$ and as a result, $V = E_\theta V_\theta = \pi V_H + (1-\pi) V_L$.

Notice that upon having received a high price offer, the posterior beliefs of the customer who observes the high signal are strictly higher than the posterior beliefs of the customer who observes the low signal. This implies $b_H^*(p_H) \geq b_L^*(p_H)$ and therefore, $\delta b_H^*(p_H) + (1-\delta) b_L^*(p_H) > (1-\delta) b_H^*(p_H) + \delta b_L^*(p_H)$ if $b_H^*(p_H) \neq b_L^*(p_H)$. Thus, although in this model, there does not exist an explicit cost of signalling high quality by charging the high price, there exists an implicit opportunity cost in terms of the probability of trade. As in the standard signalling games, this opportunity cost is higher for the low quality seller than for the high quality seller. If $b_H^*(p_H) \neq b_L^*(p_H)$, then $V_H(p_H) > V_L(p_H)$, so it must be the case that the high quality seller charges the high price in equilibrium with a probability at least as high as the probability at which the low quality seller charges this price: $\phi_H^* \geq \phi_L^*$. If no customer type accepts to trade at the high price, then the sellers' optimal response is to pool on the low price. If all customer types always accept to trade at the high price, then the sellers' optimal response is to pool on the high price. This simple observation allows us to begin by stating the following lemma:

Lemma 1. *Suppose $\frac{1}{2} < \delta < 1$. An equilibrium is of one of the following types:*

(i) $\phi_H^* = \phi_L^* = 0$.

(ii) $\phi_H^* = 1$ *and* $\phi_L^* \in (0, 1]$.

Proof. In the Appendix.

The main focus of this paper consists on studying the effect of the degree of competition and signal precision on the level of consumer fraud in equilibrium. We consider the *candidate* pooling equilibria in which both types of sellers charge the low price not interesting since there is no fraud committed in such *potential* equilibria. Thus, we restrict our analysis to study only the remaining candidate equilibria. In those potential equilibria, the high quality seller always offers his item for sale at the high price while the low quality seller may pool on the high price or may randomize between both prices. Completely fraudulent behavior by the low quality seller is encompassed in pooling equilibria whilst partial honest behavior by the low quality seller is revealed in hybrid equilibria.

Definition 2. *A Fraudulent Pooling Equilibrium is an equilibrium in which both sellers pool on the high price, that is,* $\phi_H^* = \phi_L^* = 1$*, and therefore, the low quality seller displays a completely fraudulent behavior.*

5 Equilibrium Analysis

Before embarking on the equilibrium analysis under competition, we proceed to analyze the "monopoly" problem which provides a convenient reference point.

5.1 Monopoly Regime

The monopoly regime corresponds to the case $n = 1$ in the present model. Because there is only one seller in the market, this seller will be matched with a buyer at every period of time, independently of whether a transaction took place in the previous period. Thus, the nature of the seller's problem becomes static and it suffices to calculate his per-period profits to solve for his optimal strategy.

Consequently, the analysis of the conditions for the existence of SSPBE is equivalent to the analysis of conditions for the existence of a PBE in the one seller-one buyer-one period model.

By Bayes consistency, the buyer's posterior beliefs after receiving signal s and a high price offer, conditional on her expectations about the level of honesty in the market are:

$$\mu_b(H|p_H, s) = \begin{cases} \frac{\pi\delta}{\pi\delta+(1-\pi)(1-\delta)\phi_L} \equiv \hat{\mu}_H(\phi_L) & \text{if } s = H \\ \frac{\pi(1-\delta)}{\pi(1-\delta)+(1-\pi)\delta\phi_L} \equiv \hat{\mu}_L(\phi_L) & \text{if } s = L \end{cases} \tag{6}$$

Figure 2 displays the buyer's posterior beliefs as a function of the signal precision. Due to the informativeness of the signal, the posterior beliefs upon receiving a high signal are increasing in the value of δ while the posterior beliefs upon receiving a low signal are decreasing in δ. Both posterior beliefs are decreasing in the cheating probability exerted by the low quality seller, ϕ_L. Ceteris paribus, the lower is ϕ_L, the higher is the level of honesty in the market, and the more optimistic are both types of customers about the quality of the good conditional on having been offered the item at the high price. Hence, the higher the expected valuation of the good and the more eager are both customers to get the object.

Let δ_L and δ_H denote the signal accuracy levels at which the low and high signal customers are respectively indifferent between accepting or rejecting a high price offer under outright cheating, defined as $\phi_L = 1$. Therefore, if $p_H < \bar{v}$, $\delta_L = 1 - \left(1 + \left(\frac{\pi}{1-\pi}\right)\left(\frac{v_H - p_H}{p_H - v_L}\right)\right)^{-1} \equiv 1 - \left(1 + \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-A}{A}\right)\right)^{-1}$ while the high signal customer cannot be made indifferent since then $A < \pi < \hat{\mu}_H(1)$. Alternatively, if $p_H > \bar{v}$, $\delta_H = 1 - \delta_L$, while the low signal customer cannot be made indifferent since then $\hat{\mu}_L(1) < \pi < A$. Figure 3 shows how δ_L is determined graphically.

We proceed now to establish conditions for the existence of pooling and hybrid equilibria involving consumer fraud.

Proposition 1. *Assume* $\frac{1}{2} < \delta < 1$ *and* $p_H \leq \bar{v}$.

(i) *Pooling Equilibria in which both sellers charge the high price exist if and only if* $\delta \leq \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$.

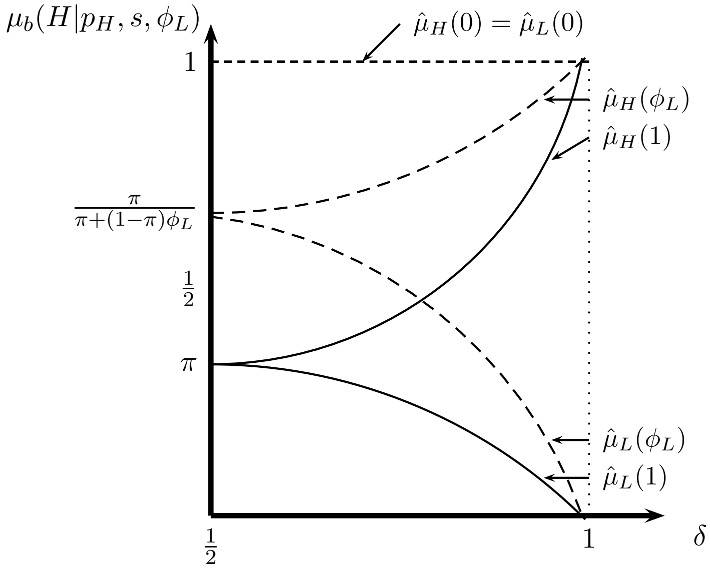

Figure 2: Buyer's Posterior Beliefs as a function of the signal precision when $\pi < \frac{1}{2}$.

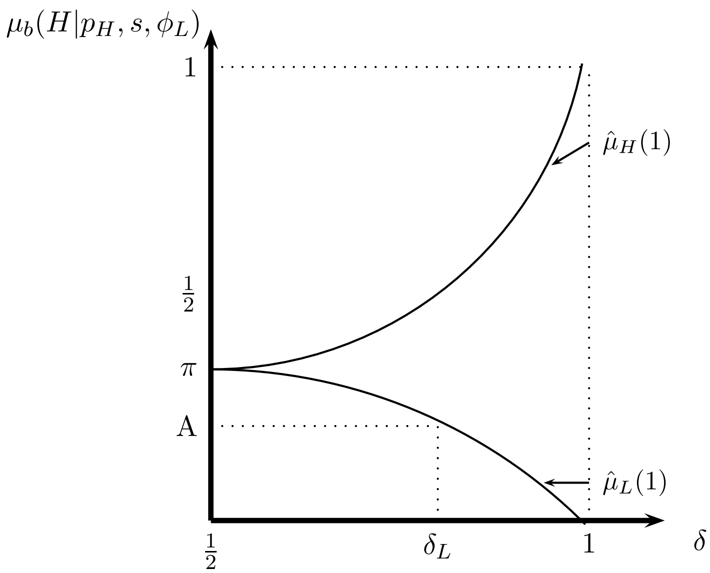

Figure 3: δ_L when $p_H < \bar{v}$ and $\pi < \frac{1}{2}$.

(ii) *Hybrid Equilibria in which the high quality seller chooses* p_H *and the low quality seller randomizes between* p_H *and* p_L *exist if and only if* $\delta \geq \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$.

Proof. In the Appendix.

Proposition 2. *Assume* $\frac{1}{2} < \delta < 1$ *and* $p_H > \bar{v}$.

(i) If $\delta_H \leq 1 - \frac{p_L}{p_H}$,

   (a) *Pooling Equilibria in which both sellers charge the high price exist if and only if* $\delta \in \left[\delta_H, 1 - \frac{p_L}{p_H}\right]$.

   (b) *Hybrid Equilibria in which the high quality seller chooses* p_H *and the low quality seller randomizes between* p_H *and* p_L *exist if and only if* $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup \left[1 - \frac{p_L}{p_H}, 1\right)$.

(ii) If $\delta_H > 1 - \frac{p_L}{p_H}$, no pooling equilibrium in which both sellers charge the high price exists while hybrid equilibria in which the high quality seller chooses p_H and the low quality seller randomizes between p_H and p_L exist $\forall \delta$.

Proof. In the Appendix.

Corollary 1. *Equilibria involving fraud exist for all* $\delta \in \left(\frac{1}{2}, 1\right)$.

Uniqueness[11]of these fraudulent pooling and hybrid equilibrium outcomes is guaranteed for almost all values of δ. For the non-generic cases $\delta = \delta_H$ and $\delta = \delta_L$, there exists a continuum of pooling equilibrium outcomes parameterized by the randomization strategy of the customer who obtains the high and low signal respectively. For the non-generic case $\delta = 1 - \frac{p_L}{p_H}$, there exists a continuum of hybrid equilibrium outcomes in accordance with the seller's randomization

[11]Recall that our analysis ignores the possible existence of pooling equilibria in which both types of sellers charge the low price as discussed above.

strategy, which is not pinned down but bounded from above and below. Specific details about the characterization of these equilibrium outcomes are provided in the appendix. We use the help of Figure 4 to illustrate the intuition behind the model. We must distinguish three possible cases depending on the value of the high price:

- Case 1 $p_H < \bar{v}$: The high price is lower than the ex ante expected valuation of the good. If the signal is fairly noisy, $\delta < \delta_L$, a low signal does not convey a negative concomitant meaning in terms of quality. The posterior beliefs of the low signal customer are updated slightly downwards so that her expected valuation, although lower than $\bar{v}$, is still greater than the high price. Hence, it is rational for both customer types to purchase the item on sale at the high price even in the most pessimistic scenario of completely fraudulent behavior. As a result, the best response of the seller to the buyer's strategies is to set the high price with independence of his type. Pooling on the high price emerges as the unique market equilibrium. We call this pooling equilibrium outcome *Pooling* HL. If the signal is fairly accurate, $\delta > \delta_L$, a low signal does convey bad news in terms of quality and the posterior beliefs of the low signal customer are updated sufficiently downwards that this buyer is no longer willing to accept this high price offer if outright cheating prevails in the market. Suppose $\delta_L < 1 - \frac{p_L}{p_H}$. If $\delta_L < \delta \leq 1 - \frac{p_L}{p_H}$, it still pays the low quality seller to practice complete fraud by targeting the high signal customer exclusively, since the unit profit is high[12] in spite of lower volumes of sale: $(1-\delta)p_H > p_L$. Thus, a pooling outcome in which both the high quality seller and low quality seller choose p_H, the high signal customer always purchases the good offered at both prices, and the low signal customer only purchases the good offered at the low price emerges as the unique market equilibrium outcome. We label this pooling equilibrium outcome as *Pooling* H. If the signal is accurate enough ($\delta > \max\{\delta_L, 1 - \frac{p_L}{p_H}\}$), then it is not rational for the low quality seller to be completely dishonest since this seller does not confront a high signal customer frequently enough. Thus, no fraudulent pooling equilibrium can be supported for this range of signal precision values. Instead, a unique hybrid outcome *(Hybrid* $H\bar{L}$*)* can be supported in

[12] Notice that a necessary condition for the existence of this equilibrium is $p_H > 2p_L$.

equilibrium: the high quality seller chooses p_H, the low quality seller randomizes and charges the high price with probability $\underline{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{v_H-p_H}{p_H-v_L}\right) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{1-A}{A}\right)$ while the high signal customer always accepts to trade at the high price and the customer who observes the low signal accepts to trade at the high price with probability $\underline{\gamma_{H\bar{L}}}(\delta) \equiv \frac{\delta-\left(1-\frac{p_L}{p_H}\right)}{\delta}$[13] and she always accepts to trade at the low price. Note that as $\delta \to 1$, $\underline{\phi}(\delta) \to 0$ and $\underline{\gamma_{H\bar{L}}}(\delta) \to \frac{p_L}{p_H}$. Therefore, as the signal is made arbitrarily precise, this hybrid equilibrium converges to a separating equilibrium in which the customer who observes the low signal accepts to trade at the high price with a strictly positive probability: $\frac{p_L}{p_H}$. Consequently, we have a discontinuity. This separating equilibrium outcome is absent under perfect information and therefore, it is not robust. The slightest decrease in signal precision causes this hybrid equilibrium outcome to emerge.

- Case 2 $p_H = \bar{v}$: The high price is equal to the ex ante expected valuation of the good. Notice then that $\delta_L = \delta_H = \frac{1}{2}$. Therefore, once the signal is realized, the high signal buyer is willing to purchase the object at the high price while the low signal buyer prefers to reject this high price offer in the event of outright cheating by the low quality seller. By the same argument as in Case 1, we have Pooling H equilibrium if $\delta < 1 - \frac{p_L}{p_H}$, and Hybrid $H\bar{L}$ equilibrium otherwise.

- Case 3 $p_H > \bar{v}$: The high price is higher than the ex ante expected valuation of the good. If the signal is not informative enough, $\delta < \min\left\{\delta_H, 1 - \frac{p_L}{p_H}\right\}$, then observing a high signal does not necessarily imply so good news in terms of quality. The posterior beliefs are updated slightly upwards and the expected valuation, although higher than $\bar{v}$, is still lower than the requested high price. As a result, this buyer is not willing to purchase the item on sale at the high price if the low quality seller defrauds continuously. Hence, no pooling equilibrium exists for this range of signal accuracy. By contrary, if there existed some level of honesty in the market, the customer's posterior beliefs could be updated upwards more strongly and the

[13] $\underline{\gamma_{H\bar{L}}}(\delta)$ is increasing and concave in δ. As $\delta \to 1 - \frac{p_L}{p_H}$, $\underline{\gamma_{H\bar{L}}}(\delta) \to 0$; as $\delta \to 1$, $\underline{\gamma_{H\bar{L}}}(\delta) \to \frac{p_L}{p_H}$.

expected valuation of the high signal customer could beat the price. This implies that there is room for an hybrid equilibrium (*Hybrid* $\bar{H}$) in which the high quality seller chooses p_H, the low quality seller randomizes strictly between setting both prices and charges the high price with probability $\bar{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{\delta}{1-\delta}\right)\left(\frac{v_H-p_H}{p_H-v_L}\right) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{\delta}{1-\delta}\right)\left(\frac{1-A}{A}\right)$, the high signal customer accepts the high price offer with probability $\underline{\gamma_{\bar{H}}}(\delta) \equiv \left(\frac{1}{1-\delta}\right)\frac{p_L}{p_H}$[14] and always accepts the low price offer while the low signal customer strictly prefers to reject the high price offer and accept the low price offer always. When the signal is fairly accurate, $\delta > \min\left\{\delta_H, 1-\frac{p_L}{p_H}\right\}$, a high signal does convey a very positive concomitant meaning in terms of quality. The posterior beliefs are updated upwards strongly enough that it is rational for this buyer to purchase the item on sale at the high price even in the most pessimistic scenario of completely fraudulent behavior. By a similar argument as the one described in *Case 1*, *Pooling H* can be supported in equilibrium for $\delta \in \left[\delta_H, 1-\frac{p_L}{p_H}\right]$ and *Hybrid* $H\bar{L}$ for $\delta \geq 1-\frac{p_L}{p_H}$.

5.1.1 Comparative Statics

In this section, we consider the effects of the signal precision, the high price and the prior, on the low quality seller's and the buyer's randomization strategies. The Hybrid $H\bar{L}$ equilibrium cheating probability $\underline{\phi}(\delta)$ by the low quality seller is decreasing in the value of δ. Ceteris paribus, the higher is δ, the more precise is the signal and the more pessimistic is the buyer about the quality of the good when the low signal is observed. Therefore, her expected valuation of the good is lower than before and in principle, she is not willing to purchase the object any longer. The seller anticipates this and he eventually cheats her less often in order to continue extracting this consumer's whole surplus by making her indifferent between accepting or rejecting the offer at the high price. On the other hand, the acceptance probability by the low signal customer $\underline{\gamma_{H\bar{L}}}(\delta)$ is increasing in δ. Ceteris paribus, the higher is δ, the higher is the probability that a low quality seller confronts a customer whose signal is low and therefore, the higher is the probability that his offer may end up being rejected. In order for this seller to remain indifferent and still be willing to randomize (otherwise, he would prefer to

[14] $\underline{\gamma_{\bar{H}}}(\delta)$ is increasing and convex in δ. As $\delta \to \frac{1}{2}$, $\underline{\gamma_{\bar{H}}}(\delta) \to 2\frac{p_L}{p_H}$; as $\delta \to 1-\frac{p_L}{p_H}$, $\underline{\gamma_{\bar{H}}}(\delta) \to 1$.

be honest), the low signal customer must accept the high price offer more frequently so that the probability of trade for the low quality seller remains invariant. For a given value of $\delta \in \left(\frac{1}{2}, 1\right)$, the value of the equilibrium cheating probability $\underline{\phi}(\delta)$ is decreasing in p_H and increasing in π. In an *Hybrid* $H\bar{L}$ equilibrium, the low quality seller swindles the buyer with a probability such that it makes the low signal customer indifferent between accepting or rejecting the high price offer. If the high price increases, ceteris paribus, then the low signal customer strictly prefers to reject the high price offer. The only way for the low signal customer to remain indifferent and still willing to accept this offer with a strictly positive probability, is through a lower deceitful probability played by the low quality seller. By doing so, he makes the low signal customer become more optimistic about the quality of the good conditional on the high price being charged. Thus, her posterior beliefs increase until her posterior expected valuation matches the new high price. In addition, a higher high price also increases the unit profit if the offer is accepted, and thus, the benefits from cheating. As a result, the low signal customer must accept the high price offer less often in order for the low quality seller to remain indifferent and be willing to continue randomizing. Consequently, we conclude that under Hybrid $H\bar{L}$ equilibrium, an increase in the high price alleviates the attempt of fraud by the low quality seller since it induces him to become more honest but at the same time, it aggravates the extent of fraud whenever it is committed. In other words, the higher benefits from cheating due to a higher high price moderates the effect on the cheating probability. By contrary, ceteris paribus, a higher prior increases the low signal customer's expected valuation of the good and now she strictly prefers to accept the offer at the high price. The seller knows that this customer is now more eager to get the good and continues to extract her entire surplus by cheating her more frequently. Consequently, we conclude that under Hybrid $H\bar{L}$ equilibrium, a higher prior exacerbates the unethical behavior of the seller.

Proposition 3. *Assume* $p_H \leq \max\{\bar{v}, 2p_L\}$*. The level of dishonesty is non-increasing in the level of signal precision in equilibrium.*

The Hybrid $\bar{H}$ equilibrium cheating probability $\bar{\phi}(\delta)$ by the low quality seller is increasing in the value of δ. The more informative the signal, the more optimistic about the quality of the object is the customer who observes the high signal, and therefore, the higher her expected valuation. This customer is now more eager to purchase the good. The low quality seller rationally knows this and he takes advantage of this situation by attempting to swindle the high signal buyer more frequently. By doing so, this buyer's posterior beliefs decrease until they reach the original value A so that the seller manages to extract the entire consumer surplus of this customer type. Similarly, the more informative the signal, the higher the probability that a given customer observes a low signal when the quality of the item is low, and the higher the probability that the low quality seller's high price offer is rejected. In order for the low quality seller to remain indifferent between charging both prices, the high signal customers, who are encountered less often, must accept trade at the high price more often. By the same reasoning as before, the swindling probability by the low quality seller is increasing in the prior and decreasing in the value of the high price.

Proposition 4. *Assume $p_H > \max\{\bar{v}, 2p_L\}$. The level of dishonesty is not monotonic in the level of signal precision in equilibrium. It increases as the signal becomes more precise if the signal is not informative enough $\left(\delta < \min\left\{\delta_H, 1 - \frac{p_L}{p_H}\right\}\right)$, while it decreases as the signal becomes more precise if the signal is fairly accurate $\left(\delta > 1 - \frac{p_L}{p_H}\right)$.*

The opposite effects of an increase in the signal precision on the level of market dishonesty as a function of the signal precision values are due to a change in the identity of the marginal customer. The Hybrid $\bar{H}$ equilibrium outcome exists for low signal precision values and the marginal customer is the customer who observes the high signal. The fact that this customer becomes more optimistic about the expected quality of the good as the signal is more precise, intensifies the seller's dishonest behavior. The Hybrid $H\bar{L}$ equilibrium outcome exists for high signal precision values so that the high signal customer is already captured and the low signal customer becomes the marginal customer. The fact that this customer is more pessimistic about the expected quality of the good as the signal is more precise, moderates the seller's unethical behavior.

Overall, notice that in the limiting case in which the low price is set arbitrarily small, the equilibrium cheating probability is non-decreasing in the signal accuracy level for all signal precision levels and high price values.

5.1.2 Incidence of Fraud and Welfare

The customer's equilibrium ex ante probability of becoming a victim of fraud or the incidence of fraud, can be defined as,

$$\Phi^* = (1-\pi)\phi_L^*[(1-\delta)b_H^*(p_H) + \delta b_L^*(p_H)]. \tag{7}$$

Similarly, the equilibrium ex ante expected level of fraud can be defined as the product of the incidence of fraud and the extent of fraud, $Efraud = \Phi^* \ (p_H - v_L)$.

In Pooling HL equilibrium trade always takes place between the buyer and the seller with independence of the signal received. As a result, the potential gains from trade are fully realized so that this equilibrium is efficient. In addition, the incidence of fraud remains constant at its highest possible value, $1-\pi$, as the signal becomes more precise. Therefore, the expected level of fraud is independent of the level of signal precision.

In Pooling H equilibrium, the more precise the signal, the lower the probability that a given customer observes the high signal given that the object is of low quality, and therefore, the lower the incidence of fraud (which is given by $(1-\pi)(1-\delta)$) and the expected level of fraud. This equilibrium is inefficient as the potential gains from trade are not fully realized:

$$W_{pH}^* = \pi\delta v_H + (1-\pi)(1-\delta)v_L < \bar{W}^*$$

The more precise the signal, the more optimistic about the expected quality is the buyer whose signal is high and thus, the higher her expected utility. The customer whose signal is low does not trade and her expected utility is zero. Hence, the buyer's ex ante expected utility is increasing in the signal accuracy. On the other hand, the more precise the signal, the higher the trade probability and thus, the utility, for the high quality seller and the lower for the low quality seller. Therefore, the seller's ex ante expected utility is increasing in δ if and only if $\pi > \frac{1}{2}$. Overall, the gains from trade increase as the signal becomes more accurate if and only if the prior is high enough ($\pi > \frac{v_L}{v_H + v_L}$).

In a Hybrid $H\bar{L}$ equilibrium, the acceptance probability $\underline{\gamma_{H\bar{L}}}(\delta)$ by the low signal customer makes the trade probability at the high price for the low quality seller invariant: $(1-\delta)+\delta\underline{\gamma_{H\bar{L}}}(\delta)=\frac{p_L}{p_H}$. The more precise the signal, the lower the equilibrium cheating probability by the low quality seller and hence, the lower the incidence of fraud (which is given by $(1-\pi)\frac{p_L}{p_H}\ \underline{\phi}(\delta)$) and the expected level of fraud in equilibrium. This equilibrium is inefficient as the potential gains from trade are not fully realized:

$$W^*_{hH\bar{L}}=\pi\left[1-\left(\frac{1-\delta}{\delta}\right)\left(1-\frac{p_L}{p_H}\right)\right]v_H+(1-\pi)\left[1-\underline{\phi}(\delta)\left(1-\frac{p_L}{p_H}\right)\right]v_L<\bar{W}^*$$

The more precise the signal, the more optimistic about the expected quality of the good is the buyer whose signal is high and who is offered the item at the high price. Thus, the more accurate the signal, the higher the expected utility of this customer. The low signal customer who is offered the item at the high price, is indifferent between accepting and rejecting this offer so that her expected utility is zero. Any customer who is offered the item at the low price obtains a strictly positive utility which is independent of the level of δ. However, the higher δ, the more often the low quality seller charges the low price. Overall, the buyer's ex ante expected utility is increasing in the level of signal precision. The low quality seller is indifferent in equilibrium between charging the high price and the low price. His utility is constant thereby and equal to the low price. The more precise the signal, the more often does the low signal customer accept the high price offer and the more often, does the high quality seller encounter a high signal customer. As a result, the high quality seller's probability of trade, and thus his expected utility, is increasing in the signal accuracy level. Given this, the more precise the signal, the higher the seller's ex ante expected utility in equilibrium. Overall, the gains from trade are increasing and concave in the signal precision. In the limit, as the signal is made arbitrarily precise, they converge to the potential gains from trade.

In a Hybrid $\bar{H}$ equilibrium, the acceptance probability $\underline{\gamma_{\bar{H}}}(\delta)$ by the high signal customer makes the trade probability at the high price for the low quality seller invariant: $(1-\delta)\underline{\gamma_{\bar{H}}}(\delta)=\frac{p_L}{p_H}$. The more precise the signal, the higher the equilibrium cheating probability by the low quality seller and hence, the higher the incidence of fraud (which is given by $(1-\pi)\frac{p_L}{p_H}\ \bar{\phi}(\delta)$) and the expected level of fraud in equilibrium. Thus, the main insight of the analysis performed in this section is into the possibility that a more precise customer's private information might lead to a higher level of fraud. This equilibrium is inefficient as the potential gains from trade are not fully realized:

$$W^*_{h\bar{H}} = \pi \left(\frac{\delta}{1-\delta}\right) \frac{p_L}{p_H} v_H + (1-\pi) \left[1 - \bar{\phi}(\delta) \left(1 - \frac{p_L}{p_H}\right)\right] v_L < \bar{W}^*$$

The buyer whose signal is high is indifferent between accepting and rejecting the high price offer. Thus, her expected utility conditional on being offered the item at the high price is zero. The low signal customer rejects all high price offers so that her utility conditional on receiving a high price offer is zero. Any customer who is offered the item at the low price obtains a strictly positive utility which is independent of the level of δ. However, the higher δ, the less often the low quality seller is honest and charges the low price. As a result, the buyer's ex ante expected utility is decreasing in the level of signal precision. The low quality seller is indifferent in equilibrium between charging the high price and the low price. His utility is constant thereby and equal to the low price. The more precise the signal, the more often does the high signal customer accept the high price offer. As a result, the high quality seller's probability of trade, and thus his expected utility, is increasing in the signal accuracy level. Given this, the more precise the signal, the higher the seller's ex ante expected utility in equilibrium. The probability of trade for both the low and high quality sellers is increasing in the low price. If the low price is high enough, a more precise signal causes the increase in the trading probability of the high quality goods dominate the decrease in the trading probability of the low quality goods which are now offered at the low price less often. Overall, the gains from trade are increasing and convex in the signal precision if and only if the low price is high enough ($p_L > (1-A)v_L$). Otherwise, they are decreasing and concave in δ.

5.2 Competition Regime

We model competition by allowing the number of sellers in the market, n, to be greater than one. The main difference between this section and the previous one is that competition makes the seller's problem dynamic. When choosing his optimal strategy, the seller now considers the effect of his current price decision on his present expected discounted value of lifetime earnings, not only his current period profits. This critical difference causes multiple equilibria to emerge for a middle range of signal precision values $\delta \in \left(\max\{\delta_L, \delta_s\}, 1 - \frac{p_L}{p_H}\right)$ and market competition levels $n \in (\underline{n}, \max\{\bar{n}_p, \bar{n}_h\})$. Under the monopoly regime, only pooling equilibria could be supported for this range of signal precision values, since the benefit of cheating was positive while its long-run cost was null. Now, instead, the long-run cost of cheating is positive due to the possibility of becoming unmatched if the high price offer is rejected by the customer. Multiplicity is interesting because it implies that two otherwise identical markets may end up being characterized by very different sellers' ethics, which eventually leads to very different levels of consumer fraud. Figure 5 illustrates the coexisting equilibrium cheating probabilities as a function of the market competition level for a fixed intermediate signal precision level. For the remaining signal precision values and all market competition levels, the "same" equilibrium outcomes that could be supported under the monopoly regime prevail under the competition regime. The only arising discrepancy refers to the acceptance probabilities by the marginal customers under hybrid equilibria. These acceptance probabilities are now deterministic functions of the level of competition in the market place. Competition results in a higher equilibrium acceptance probability by the marginal customer than monopoly does.

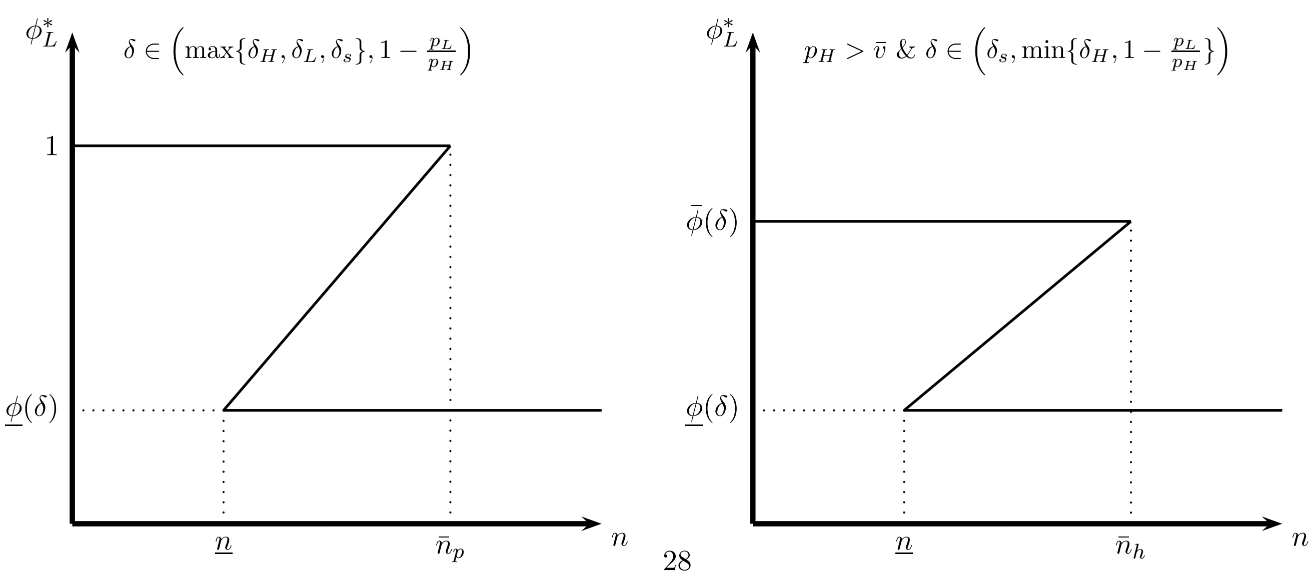

Figure 5: The Equilibrium Cheating Probability as a Function of the Market Competition Level.

In addition, the marginal customer must accept the high price offer neither too occasionally nor too often in equilibrium . If she rejected the offer too often, the low quality seller would strictly prefer to be deviate and charge only the low price while if the high price offer were accepted too often, he would strictly prefer not to randomize and be totally dishonest by charging only the high price.

Let us define:

$$\delta_s \equiv \frac{[1-\beta(1-\pi)]\left(1-\frac{p_L}{p_H}\right)}{1+\beta(2\pi-1)\left(1-\frac{p_L}{p_H}\right)} \quad ; \quad \delta_h \equiv \pi\delta + (1-\pi)(1-\delta)$$

The following propositions establish conditions for the existence of pooling and hybrid equilibria involving consumer fraud.

Proposition 5. *Assume* $\frac{1}{2} < \delta < 1$ *and* $p_H \leq \bar{v}$.

(i) *Fraudulent Pooling Equilibria exist for all possible levels of competition in the market if* $\delta \leq \max\{\delta_s, \delta_L\}$. *For any* δ, *with* $\max\{\delta_s, \delta_L\} < \delta < 1 - \frac{p_L}{p_H}$, *there exists* $\bar{n}_p$ *such that if* $n \leq \bar{n}_p$, *fraudulent pooling equilibria exist. No fraudulent pooling equilibrium exists for any other possible combination of* δ *and* n.

(ii) *Hybrid Equilibria exist for all possible levels of competition in the market if* $\delta \in \left[1 - \frac{p_L}{p_H}, 1\right)$. *For any* δ, *with* $\max\{\delta_s, \delta_L\} < \delta < 1 - \frac{p_L}{p_H}$, *there exists* $\underline{n}$ *such that if* $n \geq \underline{n}$, *hybrid equilibria exist. No hybrid equilibrium exists for any other possible combination of* δ *and* n.

Proof. In the Appendix.

Proposition 6. *Assume* $\frac{1}{2} < \delta < 1$ *and* $p_H > \bar{v}$.

(i) If $\delta_H \leq 1 - \frac{p_L}{p_H}$,

(a) *Fraudulent Pooling Equilibria exist for all possible competition levels in the market if* $\delta \in [\delta_H, \delta_s]$. *Fraudulent pooling equilibria also exist for any* δ *with* $\max\{\delta_H, \delta_s\} < \delta < 1 - \frac{p_L}{p_H}$

if $n \leq \bar{n}_p$. For any δ such that $\delta = \delta_H > \delta_s$, there exists $\bar{n}_{p4}$ such that if $n \leq \bar{n}_{p4}$, fraudulent pooling equilibria exist. No fraudulent pooling equilibrium exists for any other possible combination of δ and n.

(b) *Hybrid Equilibria exist for all possible competition levels in the market if $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup \left(1 - \frac{p_L}{p_H}, 1\right)$. Hybrid equilibria exist for any δ with $\max\{\delta_H, \delta_s\} < \delta \leq 1 - \frac{p_L}{p_H}$ if $n \geq \underline{n}$. No hybrid equilibrium exists for any other possible combination of δ and n.*

(ii) If $\delta_H > 1 - \frac{p_L}{p_H}$, no fraudulent pooling equilibrium exists and hybrid equilibria exist $\forall \delta$ & $\forall n$.

Proof. In the Appendix.

Corollary 2. *Equilibria involving fraud exist for all market competition levels and all $\delta \in \left(\frac{1}{2}, 1\right)$.*

5.2.1 The Competition Effect on Dishonesty

In this section, we consider the effect of an increase in the market competition level on the equilibrium cheating probabilities and the buyer's randomization strategy.

Proposition 7. *Assume $\delta \in \left(\max\{\delta_L, \delta_s\}, 1 - \frac{p_L}{p_H}\right)$. The level of dishonesty is non-decreasing in the level of market competition in all equilibria.*

In the left-hand side diagram of Figure 5, Pooling H is an equilibrium outcome for not very relatively highly competitive markets. Under this equilibrium outcome, the *relative* short-run benefit from cheating is $(1-\delta)p_H - p_L > 0$, while its relative long-run cost is $\beta\delta\left(1 - \frac{1}{n}\right)(V - V_u)_{pH}$. For this range of signal precision values, the following condition is satisfied: $(1-\delta)p_H - p_L < \beta\delta(V - V_u)_{pH} \equiv \beta\delta\frac{\delta_h p_H}{1-\beta\delta_h}$. Thus, if the market were relatively highly competitive, the probability of recovering a lost relationship would be so low that the long-run cost of cheating would outweigh its short-run benefit, inducing the low quality seller to deviate and be honest.

An increase in the market competition level n, has no effect on the present value of the expected future rents from remaining matched relative to unmatched, $(V - V_u)_{pH}$, nor on the rejection probability of a high price offer. However, it increases the probability that the match will not be recovered if the offer is eventually rejected. As a result, the long-run cost of cheating in the current period increases and honesty becomes relatively more attractive than before. If the short-run benefit from cheating are large enough that they still outweigh the increased long-run cost of cheating, then pooling on the high price remains optimal.

In any hybrid equilibrium, the low quality seller must be indifferent between being honest or dishonest. As a result, the relative short-run benefit of cheating must balance its long-run cost. The incorporation of competition causes a "new"[15] hybrid equilibrium outcome to emerge for non-generic values of δ. Under this equilibrium outcome, which we call Hybrid H, the high quality seller chooses p_H, the low quality seller randomizes between p_H and p_L and cheats his customers with sufficiently high probability, ϕ_L^*, that the high signal customer always purchases the good offered at both prices, and the low signal customer only purchases the good offered at the low price. The customer's posterior beliefs must be given by equation (6) by Bayes consistency. For the buyer's strategy to be optimal, it must be that $\mu_b^*(H|p_H, L) < A < \mu_b^*(H|p_H, H)$, and therefore, $\phi_L^* \in (\underline{\phi}(\delta), \bar{\phi}(\delta))$. As in the monopoly regime, the buyer's optimality condition does not pin down the seller's dishonesty level in equilibrium. It only determines an upper-bound and lower-bound for its value. The present value of lifetime expected utilities for a matched type H and a matched type L sellers who charge the high price are respectively:

$$V_H(p_H) = \delta(p_H + \beta V) + (1-\delta)\beta\left(\frac{1}{n}V + \left(1 - \frac{1}{n}\right)V_u\right)$$

$$V_L(p_H) = (1-\delta)(p_H + \beta V) + \delta\beta\left(\frac{1}{n}V + \left(1 - \frac{1}{n}\right)V_u\right)$$

Charging the low price once yields:

$$V_H(p_L) = V_L(p_L) = p_L + \beta V$$

[15] A continuum of this equilibrium outcome exists for $\delta = 1 - \frac{p_L}{p_H}$ under the monopoly regime

In equilibrium, a one shot deviation must be not profitable for the high quality seller while the type L seller must be indifferent between charging either price. Since $V_H(p_H) > V_L(p_H)$, this condition can be written as,

$$(1-\delta)p_H - p_L = \beta\delta\left(1-\frac{1}{n}\right)(V-V_u)_{hH} \tag{8}$$

In addition, in equilibrium,

$$V = \pi V_H(p_H) + (1-\pi)\phi_L^* V_L(p_H) + (1-\pi)(1-\phi_L^*)V_L(p_L)$$

which can be written as:

$$V = [\pi\delta+(1-\pi)(1-\delta)\phi_L^*](p_H+\beta V)+(1-\pi)(1-\phi_L^*)(p_L+\beta V)+[\pi(1-\delta)+(1-\pi)\delta\phi_L^*]\beta\left(\frac{1}{n}V+\left(1-\frac{1}{n}\right)V_u\right)$$

Also,

$$V_u = [\pi\delta+(1-\pi)(1-\delta)\phi_L^*+(1-\pi)(1-\phi_L^*)]\beta V_u + [\pi(1-\delta)+(1-\pi)\delta\phi_L^*]\beta\left(\frac{1}{n}V+\left(1-\frac{1}{n}\right)V_u\right)$$

so that

$$(V-V_u)_{hH} = \frac{[\pi\delta+(1-\pi)(1-\delta)\phi_L^*]p_H+(1-\pi)(1-\phi_L^*)p_L}{1-\beta[\pi\delta+(1-\pi)(1-\delta)\phi_L^*+(1-\pi)(1-\phi_L^*)]} \tag{9}$$

Equations (8) and (9) can be solved for ϕ_L^*:

$$\phi_L^* = \frac{\delta-(1-\beta\delta_h)\left(1-\frac{p_L}{p_H}\right)-\frac{\beta\delta}{n}[\pi\delta+(1-\pi)\frac{p_L}{p_H}]}{\frac{\beta(1-\pi)\delta}{n}\left(1-\frac{p_L}{p_H}-\delta\right)} \tag{10}$$ [16]

A necessary condition for Hybrid H to be an equilibrium is that competition cannot be either relatively too intense nor too moderate. If competition were relatively too intense, the probability of

[16]For the relevant range of signal precision values, the following can be shown: $\frac{\partial\phi_L^*}{\partial n} > 0$, $\frac{\partial\phi_L^*}{\partial\delta} > 0$, $\frac{\partial^2\phi_L^*}{\partial n\partial\delta} > 0$, $\frac{\partial\phi_L^*}{\partial\frac{p_L}{p_H}} > 0$, $\frac{\partial\phi_L^*}{\partial\pi} > 0$. As $n \to \underline{n}$, $\phi_L^* \to \underline{\phi}(\delta)$; as $n \to \bar{n}_h$, $\phi_L^* \to \bar{\phi}(\delta)$.

recovering a lost relationship would be low and hence, the short-run benefit from cheating would not compensate for the expected potential gains that could be foregone if the seller became unmatched. If competition were relatively moderate, and the buyer did not accept the high price offer, the seller would be rematched with the buyer with a relatively high probability. Thus, the probability of becoming an unmatched seller would be low and hence, the short-run benefit from cheating in the current period would exceed its long-run cost. The Hybrid H equilibrium cheating probability is increasing in the level of competition in the market. The intuition is as follows. An increase in the market competition level, increases the probability that the match will not be recovered if the offer is eventually rejected. As a result, the long-run cost of cheating in the current period increases and honesty becomes relatively more attractive than before. Notice that a matched seller who charged the high price with probability $\phi \in (\underline{\phi}(\delta), \bar{\phi}(\delta))$ would obtain expected per-period rents of $[\pi\delta + (1-\pi)(1-\delta)\phi]p_H + (1-\pi)(1-\phi)p_L$. The higher the dishonesty level, the higher the per-period rents. However, the more frequently the low quality seller cheated his customer, the lower would be the probability of keeping the relationship alive, and thus, the shorter the expected duration of enjoying those per-period rents. Consequently, the market turnover rate would increase. For this range of signal precision values, the second effect dominates and the relative gains from being matched relative to unmatched would decrease as the low quality seller became less honest. Thus, if the low quality seller cheated his customer more frequently, the long-run cost of cheating in the current period would decrease. Eventually, for a sufficiently high cheating probability, the long-run cost of cheating would match the relative short-run benefit from cheating and the low quality sellers would again be indifferent between being honest and dishonest. Hence, the Hybrid H equilibrium cheating probability by the low quality seller increases with competition.

The low quality seller's indifference condition in a Hybrid $H\bar{L}$ equilibrium is given by:

$$(1 - \delta + \delta\gamma_{hH\bar{L}})p_H - p_L = \beta\delta(1 - \gamma_{hH\bar{L}}) \left(1 - \frac{1}{n}\right) (V - V_u)_{hH\bar{L}} \tag{11}$$

where $\gamma_{hH\bar{L}} \equiv b^*_L(p_H)_{hH\bar{L}}$ and it can be shown that

$$(V - V_u)_{hH\bar{L}} = \frac{(\pi[\delta + (1-\delta)\gamma_{hH\bar{L}}] + (1-\pi)\underline{\phi}(\delta)[(1-\delta) + \delta\gamma_{hH\bar{L}}])p_H + (1-\pi)(1-\underline{\phi}(\delta))p_L}{1 - \beta(\pi[\delta + (1-\delta)\gamma_{hH\bar{L}}] + (1-\pi)\underline{\phi}(\delta)[(1-\delta) + \delta\gamma_{hH\bar{L}}] + (1-\pi)(1-\underline{\phi}(\delta)))} \tag{12}$$

Equations (11) and (19) determine $\gamma_{hH\bar{L}}$ in equilibrium. A necessary condition for Hybrid $H\bar{L}$ to be an equilibrium is that competition cannot be relatively too moderate and that the low signal customer cannot accept the offer too often. If competition were relatively moderate, and the buyer did not accept the high price offer, the seller would be rematched with the buyer with a relatively high probability if the high price offer were rejected. Thus, the probability of becoming an unmatched seller would be low and hence, the short-run benefit from cheating in the current period would exceed its long-run cost. If the low signal customer were to accept the high price offer too often, then the probability of rejection would be very low so that the short-run benefit from cheating more than compensates its long-run cost and the low quality seller would have an incentive to deviate and outright cheat. Competition could be relatively intense. If that were the case, then the probability of acceptance of the high price offer by the low signal customer must be high. Specifically, in equilibrium, both the market competition level and the acceptance probability move in the same direction. That is, $\gamma_{hH\bar{L}}$ is increasing in n, or alternatively, solving for n as a function of γ, it can be shown that $n_{hH\bar{L}}(\delta, \gamma)$ is increasing in γ. An increase in the market competition level, increases the long-run cost of cheating in the current period and honesty becomes relatively more attractive than before. In equilibrium, however, the low quality seller must cheat his customer with the same frequency as previously. Otherwise the low signal customer would no longer be indifferent between accepting or rejecting the high price offer. A higher acceptance by the low signal customer increases both the expected per-periods rents, that would be obtained if the relationship is kept alive, and the expected duration of keeping it alive. Consequently, the actual gains from being matched relative to unmatched increase. Nonetheless, it also decreases the probability of losing the relationship by cheating in the current period. Overall, if the low signal customer were to accept the high price with a sufficiently higher probability than before, the short-benefit from cheating could increase by more than the increase in its long-run cost and this effect could just be sufficient to compensate for the initial effect caused by the increase in the market competition level. Thereby, the low quality seller could continue cheating his potential customer with the same frequency and still remain indifferent among both options.

Finally, the following condition must be satisfied in a Hybrid $\bar{H}$ equilibrium:

$$(1-\delta)\gamma_{h\bar{H}}p_H - p_L = \beta[\delta + (1-\delta)(1-\gamma_{h\bar{H}})]\left(1-\frac{1}{n}\right)(V-V_u)_{h\bar{H}} \tag{13}$$

where $\gamma_{h\bar{H}} \equiv b^*_H(p_H)_{h\bar{H}}$ and

$$(V-V_u)_{h\bar{H}} = \frac{[\pi\delta + (1-\pi)(1-\delta)\bar{\phi}(\delta)]\gamma_{h\bar{H}}p_H + (1-\pi)(1-\phi)p_L}{1-\beta\{[\pi\delta + (1-\pi)(1-\delta)\bar{\phi}(\delta)]\gamma_{h\bar{H}} + (1-\pi)(1-\bar{\phi}(\delta))\}} \tag{14}$$

A necessary condition for Hybrid $\bar{H}$ to be an equilibrium is that competition cannot be relatively too intense and that the low signal customer cannot accept the offer too occasionally. If competition were relatively too intense, the low quality seller would have an incentive to deviate and be honest due to the low probability of being rematched with the buyer in the following period if the high price offer were rejected. If the high signal customer were to accept the high price offer too occasionally, then the probability of rejection would be very high so that the short-run benefit from cheating would be below its long-run cost.

As mentioned above, ceteris paribus, an increase in the market competition level increases the long run cost of cheating. In equilibrium, however, the low quality seller must cheat his customer with the same frequency as previously in order for the high signal customer to remain indifferent between accepting or rejecting the high price offer. A higher acceptance by the low signal customer increases both the expected per-periods rents, that would be obtained if the relationship is kept alive, and the expected duration of keeping it alive. Consequently, the actual gains from being matched relative to unmatched increase. Nonetheless, it also decreases the probability of losing the relationship by cheating in the current period. Suppose first that the signal is not too precise or if it happens to be too precise, that the high signal customer does not reject the high price offer too occasionally. Then, if the high signal customer were to accept the high price with a sufficiently higher probability than before, the short-benefit from cheating could increase by more than the increase in its long-run cost and this effect could just be sufficient to offset the initial effect caused by the increase in the market competition level. In the remaining cases, if the high signal customer were to accept the high price with a sufficiently higher probability than before, the short-benefit from cheating would increase by less than the increase in its long-run cost. As a result, a lower acceptance probability than before is necessary to counteract the initial effect caused by the increase in the market competition level.

5.2.2 The Competition Effect on The Incidence of Fraud

In this section we analyze the impact of changes in the market competition level on the customer's ex ante probability of becoming a victim of fraud and hence, on the expected level of fraud in the market.

Under Pooling H equilibrium, neither the low quality seller's cheating probability nor the buyer's strategy depend on the market competition level. Thus, the incidence of fraud is also independent on the market competition level and it is given by $(1-\pi)(1-\delta)$. As in the monopoly case, as the signal becomes more precise, the low quality seller's probability of trade at the high price, which is given by the probability that the low quality seller encounters a high signal customer, decreases, lowering the incidence of fraud. In Hybrid H equilibrium, however, the low quality seller cheats more frequently as competition increases. The incidence of fraud is given by $(1-\pi)(1-\delta)\phi_L^*$ and it is therefore, increasing in the market competition level. Note that for a given market competition level, the incidence of fraud is lower under Hybrid H equilibrium than under Pooling H equilibrium because the low quality seller is strictly honest occasionally in the former equilibrium. An increase in the signal precision, induces the low quality seller to cheat more frequently in equilibrium but it also lowers the trade probability. It turns out that the first effect dominates the second effect and the incidence of fraud increases as the signal becomes more precise. In Hybrid $H\bar{L}$ equilibrium, the low quality seller swindles his potential customer with a probability that is independent of the market competition level. Nonetheless, the low signal customer accepts to trade at the high price more often in highly competitive markets in equilibrium. As a result, the incidence of fraud, which is given by $(1-\pi)[(1-\delta)+\delta\gamma_{hH\bar{L}}]\underline{\phi}(\delta)$, is increasing in the level of competition in the market. For a given market competition level, the more precise the signal, the higher the acceptance probability of the high price offer by the low signal customer. In spite of this, the low quality seller's probability of trade at the high price decreases as the high signal customer, who always accepts the high price offer, is encountered less often. This effect together with a lower deceitful probability by the low quality seller decreases the incidence of fraud as the signal becomes more accurate. The incidence of fraud in Hybrid $\bar{H}$ equilibrium is given by: $(1-\pi)(1-\delta)\gamma_{h\bar{H}}\bar{\phi}(\delta)$. The frequency by which the low quality seller deceives his potential customer in Hybrid $\bar{H}$ equilibrium is independent of the market competition level as in the previous equilibrium. As a result, an increase in the market competition level increases the incidence of fraud if and only if the increase in competition induces the high signal customer to accept the high price offer more frequently. This is satisfied whenever the signal is not too precise or if it happens to be too precise, whenever the high signal customer does not reject the high price offer too occasionally. The increase in the market competition level decreases the incidence of fraud otherwise. In addition, an increase in the accuracy of the signal, increases the incidence of fraud indisputably if for a given competition level, the increase in the signal accuracy, increases the acceptance probability by the high signal customer. Otherwise, a more precise signal increases the incidence of fraud if the increase in the precision of the signal does not decrease the acceptance probability by the high signal customer by more than $(\frac{\gamma_{h\bar{H}}}{\delta})$.

To conclude, it is worth emphasizing that although a higher competition level tends to increase the probability at which an ex ante potential customer becomes a victim of fraud, the analysis from the next section reveals that under Hybrid $H\bar{L}$ and Hybrid $\bar{H}$ equilibria, the ex ante customer's expected utility is not affected by the change in the competition level. This is because the equilibrium level of honesty in the market is independent of the competition level in the market. Furthermore, it is also worth mentioning that the incorporation of competition into the market could a priori decrease the existing incidence of fraud level if a switch from one equilibrium into another equilibrium develops. For instance, this is always the case in Case 1, after a move from Pooling H equilibrium into Hybrid H equilibrium or after a move from Pooling H equilibrium into Hybrid $H\bar{L}$ equilibrium. Thus, considering only the previously mentioned effect may lead one to incorrectly arrive at an unduly pessimistic conclusion about the entire impact of competition on the incidence of fraud. Identifying competition as a source of multiple equilibria is crucial in this respect.

5.2.3 The Competition Effect on Welfare

In this section, we study the effect of changes in the market competition level on welfare and draw a comparison of the welfare results among the multiple equilibria. Let us define:

$$\delta_u \equiv \frac{1-\beta(1-\pi)}{1+\beta(2\pi-1)}; \quad \delta_t \equiv \frac{1-\beta(1-\pi)\left(1-\frac{p_L}{p_H}\right)}{1+\beta(2\pi-1)\left(1-\frac{p_L}{p_H}\right)}$$

where $\delta_s < \delta_u < \delta_t$.

Proposition 8. *Assume* $\delta \in \left(\max\{\delta_L, \delta_s\}, 1-\frac{p_L}{p_H}\right)$. *The following can be shown:*

(i) Pooling H equilibrium: $\frac{d(u_b^*)_{pH}}{dn} = 0$; $\frac{dW^*_{pH}}{dn} = 0$; $\frac{d(V^*)_{pH}}{dn} < 0$; $\frac{d(V_u^*)_{pH}}{dn} < 0$

(ii) Hybrid H equilibrium: $\frac{d(u_b^*)_{hH}}{dn} < 0$; $\frac{dW^*_{hH}}{dn} < 0$; $\frac{d(V^*)_{hH}}{dn} = 0$; $\frac{d(V_u^*)_{hH}}{dn} > 0$

(iii) Hybrid $H\bar{L}$ equilibrium: $\frac{d(u_b^*)_{hH\bar{L}}}{dn} = 0$; $\frac{dW^*_{hH\bar{L}}}{dn} > 0$; $\frac{d(V^*)_{hH\bar{L}}}{dn} = 0$; $\frac{d(V_u^*)_{hH\bar{L}}}{dn} < 0$

(iv) Hybrid $\bar{H}$ equilibrium:

(a) For any δ with $\delta \in (\delta_u, \delta_t)$ there exists $\gamma_{h\bar{H}1}$, and for any δ with $\delta > \delta_t$ there exists $\gamma_{h\bar{H}2}$, such that if $\delta < \delta_u$ or if $\delta \in (\delta_u, \delta_t)$ & $\gamma_{h\bar{H}} < \gamma_{h\bar{H}1}$ or if $\delta > \delta_t$ & $\gamma_{h\bar{H}} < \gamma_{h\bar{H}2}$[17]: $\frac{d(u_b^*)_{h\bar{H}}}{dn} = 0$; $\frac{dW^*_{h\bar{H}}}{dn} > 0$; $\frac{d(V^*)_{h\bar{H}}}{dn} > 0$; $\frac{d(V_u^*)_{h\bar{H}}}{dn} < 0$

(b) Otherwise: $\frac{d(u_b^*)_{h\bar{H}}}{dn} = 0$; $\frac{dW^*_{h\bar{H}}}{dn} < 0$; $\frac{d(V^*)_{h\bar{H}}}{dn} < 0$; $\frac{d(V_u^*)_{h\bar{H}}}{dn} > 0$.

In general, the impact of competition on the welfare variables of our interest depends specifically on the equilibrium type considered.

[17] Note that if $\gamma_{h\bar{H}1}$ or $\gamma_{h\bar{H}2}$ were lower than 1 for some fixed δ, then Hybrid $\bar{H}$ and Hybrid $H\bar{L}$ would also coexist for some range of market competition levels greater than $\bar{n}_h$.

In a Pooling H equilibrium, given that the level of honesty is not affected by a change in the market competition level, the buyer's ex ante expected utility is independent of n and so are the gains from trade. The probability of recovering a lost relationship is now lower than before so that the ex ante matched seller becomes worse off. Likewise, the probability of being randomly matched at any period decreases and the ex ante unmatched seller is made worse off too.

In a Hybrid H equilibrium, the increase in competition induces the low quality seller to cheat his potential customer more often. Thus, both the customer who observes the high signal and purchases the item at hand, and the customer who is offered the item at the low price, are made worse off. This results in a decrease in the ex ante expected utility of the buyer. Nonetheless, the ex ante customer is better off under Hybrid H than under Pooling H because in the former equilibrium, the low quality seller is strictly honest occasionally[18]. From the sellers' perspective, an increase in competition has the following effects: (1) it decreases the probability of becoming randomly matched (or of recovering the relationship) at any period of time; the increase in the cheating probability by the low quality seller (2) decreases the gains from being matched relative to unmatched and (3) increases the ex ante probability that the offer is rejected at any period of time, thereby increasing the market turnover rate. Overall, the last effect dominates the former two effects so that the unemployed seller is made better off:

$$(V_u)_{hH} = \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{n}\right)[\pi(1-\delta) + (1-\pi)\delta\phi_L^*](V - V_u)_{hH}(\phi_L^*)$$

is increasing in n. On the other hand, the effects are such that the ex ante (and ex post) matched seller is equally off. To see this, suppose a priori that there were no chances of losing the relationship if the offer were rejected, as in the monopoly case. Then, the ex ante matched seller would make an expected rent of $[\pi\delta + (1-\pi)(1-\delta)\phi_L^*]p_H + (1-\pi)(1-\phi_L^*)p_L$ in every future period of time. The higher cheating probability increases the per-period expected rents. However, we must acknowledge that with competition, the relationship may be broken up and not recovered at some particular time. If that were the case, the matched seller would become unmatched and hence, he would miss

[18] In the limit, as $n \to \bar{n}_p$, $(u_b^*)_{hH} \to (u_b^*)_{pH}$.

the actual gains from being matched relative to unmatched:

$$(V)_{hH} = \left(\frac{1}{1-\beta}\right)\left\{\delta_h(\phi_L^*)p_H + (1-\pi)(1-\phi_L^*)p_L - \beta\left(1-\frac{1}{n}\right)\delta_l(\phi_L^*)(V-V_u)_{hH}(\phi_L^*)\right\}$$

where $\delta_h(\phi_L^*) \equiv \pi\delta + (1-\pi)(1-\delta)\phi_L^*$ and $\delta_l(\phi_L^*) \equiv \pi(1-\delta) + (1-\pi)\delta\phi_L^*$

By equation (8), the above condition can be rewritten as:

$$(V)_{hH} = \left(\frac{1}{1-\beta}\right)\left\{p_L + \pi\left(\frac{2\delta-1}{\delta}\right)(p_H - p_L)\right\}$$

which is independent of n. Hence, the increase in the lifetime value of being unemployed exactly offsets the decrease in the gains from being matched relative to unmatched so that the ex ante matched seller is equally off: $V = (V_u)_{hH} + (V - V_u)_{hH}$. Notice that whenever the low quality seller is honest, trade occurs independently of the signal realization. Consequently, as the market competition level increases, the low quality seller becomes more dishonest and hence, the gains from trade decrease.

In a Hybrid $H\bar{L}$ equilibrium, the increase in competition has no effect on the low quality seller's ethical behavior and therefore, it has no impact on the high signal customer's expected utility. However, it induces the low signal customer to accept the high price offer more often. In equilibrium, this customer type must remain indifferent between accepting or rejecting the high price offer though. Therefore, the increase in competition has no effect on the ex ante expected utility of the buyer. However, the higher trade probability by low signal customer increases the equilibrium gains from trade. From the sellers' perspective, an increase in competition has the following effects: (1) it decreases the probability of becoming randomly matched (or of recovering the relationship) at any period of time; the increase in the acceptance probability by the low signal customer (2) increases the gains from being matched relative to unmatched and (3) decreases the ex ante probability that the offer is rejected at any period of time, thereby decreasing the market turnover rate. Overall, the first and last effect dominate the second effect so that the unemployed seller is made worse off:

$$(V_u)_{hH\bar{L}} = \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{n}\right)\delta_l(\underline{\phi})(1-\gamma_{hH\bar{L}})(V-V_u)_{hH\bar{L}}$$

is decreasing in n, where $\delta_l(\underline{\phi}) \equiv \pi(1-\delta)+(1-\pi)\delta\underline{\phi}(\delta)$. On the other hand, the effects are such that the ex ante (and ex post) matched seller is equally off. To see this, let $\delta_h(\underline{\phi}) \equiv \pi\delta+(1-\pi)(1-\delta)\underline{\phi}(\delta)$. If a priori there were no chances of losing the relationship if the offer were rejected, then, the ex ante matched seller would make an expected rent of $[\delta_h(\underline{\phi}) + \delta_l(\underline{\phi})\gamma_{hH\bar{L}}]p_H + (1-\pi)(1-\underline{\phi}(\delta))p_L \equiv ER_{hH\bar{L}}(\delta)$ in every future period of time. The higher acceptance probability by the low signal customer increases the per-period expected rents. However, once competition is incorporated, the relationship may be broken up and not recovered at some particular time. If that were the case, the matched seller would become unmatched and hence, he would miss the actual gains from being matched relative to unmatched:

$$(V)_{hH\bar{L}} = \left(\frac{1}{1-\beta}\right)\left\{ER_{hH\bar{L}}(\delta) - \beta\left(1-\frac{1}{n}\right)\delta_l(\underline{\phi}(\delta))(1-\gamma_{hH\bar{L}})(V-V_u)_{hH\bar{L}}\right\}$$

By equation (11), the above condition can be rewritten as:

$$(V)_{hH\bar{L}} = \left(\frac{1}{1-\beta}\right)\left\{p_L + \pi\left(\frac{2\delta-1}{\delta}\right)(p_H - p_L)\right\}$$

which is independent of n, and equal to the ex ante matched seller's present discounted value of lifetime earnings in Hybrid H equilibrium. Hence, the increase in the lifetime value of being unemployed exactly offsets the decrease in the gains from being matched relative to unmatched so that the ex ante matched seller is equally off: $V = (V_u)_{hH\bar{L}} + (V - V_u)_{hH\bar{L}}$.

In a Hybrid $\bar{H}$ equilibrium, the increase in competition has no effect on the low quality seller's ethical behavior but we showed previously that under certain conditions, it induces the high signal customer to accept the high price offer more often. In equilibrium, this customer type must remain indifferent between accepting or rejecting the high price offer. The low signal customer does not accept the high price offer. As a result, the increase in competition has no effect on the ex ante expected utility of the buyer but it increases the gains from trade. From the sellers' perspective, an increase in competition has the following effects: (1) it decreases the probability of becoming randomly matched (or of recovering the relationship) at any period of time; the increase in the acceptance probability by the high signal customer (2) increases the gains from being matched relative to unmatched and (3) decreases the ex ante probability that an offer is rejected at any period of time, thereby decreasing the market turnover rate. Overall, the first and last effect dominate the second effect so that the unemployed seller is made worse off:

$$(V_u)_{h\bar{H}} = \left(\frac{\beta}{1-\beta}\right)\left(\frac{1}{n}\right)(\delta_h(\bar{\phi})(1-\gamma_{h\bar{H}}) + \delta_l(\bar{\phi}))(V - V_u)_{h\bar{H}}(\bar{\phi}(\delta))$$

is decreasing in n[19], where $\delta_h(\bar{\phi}) \equiv \pi\delta + (1-\pi)(1-\delta)\bar{\phi}(\delta)$ and $\delta_l(\bar{\phi}) \equiv \pi(1-\delta) + (1-\pi)\delta\bar{\phi}(\delta)$. On the other hand, the effects are such that the ex ante (and ex post) matched seller is better off. To see this, if a priori there were no chances of losing the relationship in case an offer were rejected, then, the ex ante matched seller would make an expected rent of $\delta_h(\bar{\phi})\gamma_{h\bar{H}}p_H + (1-\pi)(1-\bar{\phi}(\delta))p_L \equiv ER_{h\bar{H}}(\delta)$ in every future period of time. The higher acceptance probability by the low signal customer increases the per-period expected rents. However, once competition is incorporated, the ex ante matched seller could become unmatched after an offer rejection and hence, he would miss the actual gains from being matched relative to unmatched:

$$(V)_{h\bar{H}} = \left(\frac{1}{1-\beta}\right)\left\{ER_{h\bar{H}}(\delta) - \beta\left(1-\frac{1}{n}\right)(\delta_h(\bar{\phi})(1-\gamma_{h\bar{H}}) + \delta_l(\bar{\phi}))(V - V_u)_{h\bar{H}}(\bar{\phi}(\delta))\right\}$$

By equation (13), the above condition can be rewritten as:

$$(V)_{h\bar{H}} = \left(\frac{1}{1-\beta}\right)\left\{p_L + \pi\left(\frac{(2\delta-1)\gamma_{h\bar{H}}}{1-(1-\delta)\gamma}\right)(p_H - p_L)\right\}$$

which is increasing in $\gamma_{h\bar{H}}$, and thus, under these certain conditions, in n. Hence, the decrease in the lifetime value of being unemployed is not sufficient to counteract the increase in the gains from being matched relative to unmatched so that the ex ante matched seller is better off.

Proposition 9. *Competition-regime expected utilities, present discounted value of lifetime earnings and gains from trade are ordered as follows:*

(i) If $\delta \in \left(\max\{\delta_H, \delta_L, \delta_s\}, 1 - \frac{p_L}{p_H}\right)$,

[19] Obviously, under the remaining conditions, the increase in competition induces the high signal customer to accept the high price offer less often. This causes a decrease in the gains from being matched relative to unmatched and an increase in the market turnover rate. The last effect dominates and as a result, the unmatched seller becomes better off.

(a) *From the ex ante customer's perspective:* $(u_b^*)_{hH\bar{L}} \geq (u_b^*)_{hH} \geq (u_b^*)_{pH}$

(b) *From per period gains from trade's perspective:* $W_{hH\bar{L}}^* \geq W_{hH}^* \geq W_{pH}^*$

(c) *From the ex ante matched seller's perspective:* $(V^*)_{pH} \geq (V^*)_{hH} = (V^*)_{hH\bar{L}}$

(d) *From the unmatched seller's perspective:* $(V_u^*)_{pH} \geq (V_u^*)_{hH} \geq (V_u^*)_{hH\bar{L}}$.

(ii) *If* $p_H > \bar{v}$ & $\delta \in \left(\delta_s, \min\{\delta_H, 1 - \frac{p_L}{p_H}\}\right)$,

(a) *From the ex ante customer's perspective:* $(u_b^*)_{hH\bar{L}} \geq (u_b^*)_{hH} \geq (u_b^*)_{h\bar{H}}$

(b) *From per period gains from trade's perspective:* $W_{hH\bar{L}}^* \geq W_{hH}^* \geq W_{h\bar{H}}^*$

(c) *From the ex ante matched seller's perspective:* $(V^*)_{hH\bar{L}} = (V^*)_{hH} \geq (V^*)_{h\bar{H}}$

(d) *From the unmatched seller's perspective:* $(V_u^*)_{h\bar{H}} \geq (V_u^*)_{hH} \geq (V_u^*)_{hH\bar{L}}$.

Under the conditions displayed in (i), Pooling H is preferred from the sellers' perspective. Note that competition is considered as not desirable in terms of the overall welfare under Pooling H equilibrium. Hybrid $H\bar{L}$ depicts itself as the most preferred equilibrium from the perspective of the customer and the per-period gains from trade. Moreover, it is also the most preferred equilibrium from the ex ante matched seller's perspective under the conditions displayed in (ii). Note that competition is considered as desirable in terms of the the per-period gains from trade under Hybrid $H\bar{L}$ equilibrium. It only has a negative impact on the unmatched seller's expected discounted lifetime utility. Hybrid $\bar{H}$ equilibrium does not depict itself as an interesting equilibrium from the perspective of the matched seller and buyer. Only the unmatched seller weakly prefers it to the other possible equilibria, and in general, competition is not desirable from the unmatched seller's perspective under Hybrid $\bar{H}$ equilibrium.

A central authority would not concerned about the specific shares obtained by each agent. The Social Planner would be interested in achieving the largest possible gains from trade and the lowest possible incidence of fraud level. She would rank the multiple equilibria according to these two criteria. For a given signal precision and market competition level, Hybrid $H\bar{L}$ depicts itself as the most efficient equilibrium in both respects.

6 Conclusions

This paper has explored the role played by the accuracy of consumers' private information and the effect of competition on the equilibrium levels of honesty and incidence of fraud in a two-sided asymmetric information model in which prices are fixed. We have shown that equilibria involving fraud exist for all parameter values. Moreover, we have argued that under certain conditions, an increase in the accuracy of the customer's private information may lead to a higher level of fraud in equilibrium. This is the case whenever the available information mechanism in the market is not very precise and the fixed high price is relatively high. Under these circumstances, it pays the low quality seller to target the customer who observes the high signal only by randomizing between the two prices and charging the high price with a sufficiently high probability. Since this customer observes the "wrong" signal, she becomes more optimistic about the expected quality of the good as the private information gets more accurate. This, in fact, increases her expected valuation of the good and induces the seller whose good is of low quality to continue extracting this customer's entire surplus by swindling his customer more often. Finally, we have argued that competition, by making the seller's problem dynamic, leads to multiple equilibria for an intermediate range of signal precision values and market competition levels. The equilibrium level of dishonesty is found to be nondecreasing in the level of competition in the market. An increase in the market competition level increases the long-run cost of cheating, so that honesty becomes relatively more attractive than before. However, honesty cannot prevail in the market place as the customer would become credulous and the seller would be tempted to outright cheat her. For the original equilibrium conditions to be restored, the relative long-run cost of cheating must decrease. In those equilibrium outcomes in which no customer type is indifferent between accepting or rejecting the high price offer, the relative long-run cost of cheating is lowered by letting the low quality seller cheat his customer more often. By doing so, the market turnover rate would be higher, increasing thereby the seller's value without a preexisting buyer and hence, reducing the gains from being matched relative to unmatched. In those equilibrium outcomes in which some customer type is indifferent between accepting or rejecting the high price offer, the equilibrium cheating probability cannot be modified for the marginal customer type to remain indifferent between both options. Instead, the long-run cost of cheating is lowered by generally letting the marginal customer accept the high price offer more often in equilibrium. Among the multiple equilibrium outcomes, the one in which the customer who observes the low signal is made indifferent between accepting or rejecting the high price offer, is the most efficient equilibrium outcome, since it involves the greatest gains from trade and the lowest incidence of fraud. A more accurate private information and higher competition level enhance welfare under this equilibrium outcome, despite the higher incidence of fraud caused by the increase in competition.

6.1 Future Extensions

There are several directions for future research. An interesting extension would be to study the impact of enhancing the customer's actions by allowing her to fire the seller at the end of the period if a high price has been charged and the buyer experiments low quality after consuming the good. Another aspect that is worth further examination is introducing some dynamics into the buyer's problem. This could be modelled by not guaranteeing the buyer a continuation match if she decided to reject an offer. A relevant extension would be to let the buyer influence her signal precision by imposing some cost to the customer for having access to an external source of information which reveals beforehand the quality of the product imperfectly. In the spirit of Bester and Ritzberger (1999), upon observing the price, each buyer could privately test for the quality of the good by paying a cost which is increasing in the degree of test precision chosen by the buyer. Another interesting issue is allowing customer heterogeneity in the signal precision. In the same vein, incorporating reputation concerns among sellers is also deemed natural. We leave all these issues for future work.

Appendix

Proof of Lemma 1. We must prove that for $\delta \in \left(\frac{1}{2}, 1\right)$ no separating equilibria nor hybrid equilibria in which the high type seller strictly randomizes between charging both prices nor hybrid equilibria in which the high type seller sets the low price while the low type seller randomizes, exist. The proof is by contradiction.

(i) Suppose that a separating equilibrium in which the high type seller sets the high price and the low type seller sets the low price, exists. Consider an information set $(p_H, s) \forall\ s \in S$ of the buyer. By consistency of believes along the equilibrium path, she believes that the item is of high quality with probability one. Since $v_H > p_H$, her optimal decision is to purchase the product for all possible signal realizations. But then, this implies that the low quality seller could increase his expected discounted lifetime earnings by charging the high price. A contradiction. Q.E.D.

(ii) Suppose that a separating equilibrium in which the high type seller sets the low price and the low type seller sets the high price, exists. Consider an information set $(p_H, s) \forall\ s \in S$ of the buyer. By consistency of believes along the equilibrium path, she believes that the item is of low quality with probability one. Since $v_L < p_H$, her optimal decision is not to purchase the product for all possible signal realizations. But then, this implies that the low quality seller could increase his expected discounted lifetime earnings by charging the low price. A contradiction. Q.E.D.

(iii) Suppose that an hybrid equilibrium in which both seller types strictly randomize between charging both prices, exists. Then the low quality seller must be indifferent between charging the high price and the low price. Thus, $V_L(p_H) = V_L(p_L)$ which by (2) implies $V_H(p_H) > V_L(p_H) = V_L(p_L)$. The high quality seller could increase his expected discounted lifetime earnings by not randomizing and charging only the high price. A contradiction. Q.E.D.

(iv) Suppose that an hybrid equilibrium in which the high type seller strictly randomizes between charging both prices and the low quality seller charges only the low price, exists. This results in a contradiction by the same argument as in (i). Q.E.D.

(v) Suppose that an hybrid equilibrium in which the high type seller strictly randomizes between charging both prices and the low quality seller charges only the high price, exists. The indifference condition for the high quality seller is $V_H(p_L) = V_H(p_H)$, which by (3) implies $V_H(p_L) = V_H(p_H) > V_L(p_H)$. The low quality seller could increase his expected discounted lifetime earnings by not charging the high price but the low price. A contradiction. Q.E.D.

(vi) Suppose that an hybrid equilibrium in which the high type seller sets the low price and the low quality seller strictly randomizes between charging both prices, exists. This results in a contradiction by the same argument as in (iii). Q.E.D.

Proof of Proposition 1. We prove this proposition through a series of lemmas.

Lemma 2 : *If $p_H < \bar{v}$, Pooling equilibria involving fraud exist if and only if $\delta \leq \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$.*

Proof. For necessity, suppose not, so that if $p_H < \bar{v}$, fraudulent pooling equilibria exist if $\delta > \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (6). Since $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$ so that the buyer's optimal strategies conditional on the signal received are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit of charging the high price is given by $(1-\delta)p_H < p_L$ due to the fact that $\delta > 1 - \frac{p_L}{p_H}$. The low quality seller would find it more profitable to deviate and charge exclusively the low price. A contradiction. We prove sufficiency through claims 2.1-2.3:

Claim 2.1 : *If $p_H < \bar{v}$, there exists a unique pooling equilibrium outcome characterized by the following strategies $\phi_H^* = \phi_L^* = 1$ and $b_H^*(p_H) = b_L^*(p_H) = 1$ if $\delta < \delta_L$.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta < \delta_L$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta < \delta_L$, $\mu_b^*(H|p_H, s) > A$ $\forall s \in S$. By (2), the buyer's strategy is rational with respect to them. Now consider any seller. Since the buyer accepts to trade at both prices always, and $p_H > p_L$, charging the high price is the best reply for the seller. Uniqueness for $\delta < \delta_L$ follows trivially.

Claim 2.2 : *If $p_H < \bar{v}$, there exists a continuum of pooling equilibrium outcomes characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = \gamma_L$ if $\delta = \delta_L$, where $\gamma_L \geq \frac{\delta - \left(1 - \frac{p_L}{p_H}\right)}{\delta}$.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta = \delta_L$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta = \delta_L$, then $\mu_b^*(H|p_H, H) > A$ and $\mu_b^*(H|p_H, L) = A$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. By charging the high price, his expected profit is given by $[(1-\delta) + \delta\gamma_L]p_H \geq p_L$. Thus, charging the high price is the best reply for the low quality seller and hence, also for the high quality seller $[\delta + (1-\delta)\gamma_L]p_H > p_L$.

Claim 2.3 : *If $p_H < \bar{v}$, there exists a unique pooling equilibrium outcome characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$ if $\delta \in \left(\delta_L, 1 - \frac{p_L}{p_H}\right]$.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \in \left(\delta_L, 1 - \frac{p_L}{p_H}\right]$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Since the buyer accepts to trade at the high price only if she observes the high signal, his expected profit of charging the high price is given by $(1-\delta)p_H \geq p_L$, charging the high price is the best reply for this seller and consequently, it is so for the high quality seller too. Uniqueness follows trivially.

This completes the proof of lemma 2.

Lemma 3 : *If $p_H = \bar{v}$, there exists a unique fraudulent pooling equilibrium outcome characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$ if and only if $\delta \in \left(\frac{1}{2}, 1 - \frac{p_L}{p_H}\right)$.*

Proof. If $p_H = \bar{v}$, then $\pi = A$ and $\delta_L = \frac{1}{2}$. The proof follows by using an argument similar as the one used in the proof of claim 2.3 .

Lemma 4 : *If $p_H \leq \bar{v}$, Hybrid Equilibria exist if and only if $\delta \geq \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$.*

Proof. For necessity, suppose not, so that if $p_H < \bar{v}$, hybrid equilibria exist if $\delta < \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (6). Since $\delta < \delta_L$, then $\mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L) > A$ $\forall \phi_L \in (0, 1)$ so that the buyer's optimal strategies are $b_H^*(p_H) = b_L^*(p_H) = 1$. Any seller's expected profit of charging the high price is given by $p_H > p_L$. The low quality seller would find it more profitable to deviate and charge exclusively the high price. A contradiction. If $p_H = \bar{v}$, then $\pi = A$ and $\delta_L = \frac{1}{2}$ so that $\forall \delta \in \left(\frac{1}{2}, 1\right)$, $\delta > \delta_L$ and therefore $\{\delta \in \left(\frac{1}{2}, 1\right) \mid \delta < \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}\} = \emptyset$. For sufficiency, consider the equilibrium outcome characterized by $\phi_H^* = 1$, $\phi_L^* = \underline{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{v_H - p_H}{p_H - v_L}\right)$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = \underline{\gamma_{H\bar{L}}}(\delta) \equiv \frac{\delta - \left(1 - \frac{p_L}{p_H}\right)}{\delta}$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \geq \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta \geq \delta_L$, then $\mu_b^*(H|p_H, H) > A = \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Since the buyer accepts to trade at the high price with probability one if she observes the high signal and with probability strictly less than one if she observes the low signal, the expected profit of charging the high price for the low quality seller is given by $[(1 - \delta) + \delta\underline{\gamma_{H\bar{L}}}(\delta)]p_H = p_L$. The low quality seller is indifferent between charging both prices and therefore, a randomization strategy is a best reply for this seller. Similarly, the expected profit of charging the high price for the high quality seller is given by $[\delta + (1 - \delta)\underline{\gamma_{H\bar{L}}}(\delta)]p_H > p_L$, so that charging only the high price is this seller's optimal response for the buyer's strategy . Uniqueness follows trivially.

This completes the proof of proposition 1. Q.E.D.

Proof of Proposition 2. We prove this proposition through a series of lemmas.

Lemma 5 : *If $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$, fraudulent pooling equilibria exist if and only if $\delta \in \left[\delta_H, 1 - \frac{p_L}{p_H}\right]$.*

Proof. For necessity, suppose not, so that if $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$, fraudulent pooling equilibria exist if $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup \left(1 - \frac{p_L}{p_H}, 1\right)$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$

must be given by equation (6). If $\delta < \delta_H$, then $A > \mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L)$ so that the buyer's optimal strategies are $b_H^*(p_H) = b_L^*(p_H) = 0$. Any seller who charged the high price would see his offer rejected and therefore, a low quality seller would be better off by being honest and charging only the low price. If $\delta \in \left(1 - \frac{p_L}{p_H}, 1\right)$, $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$, so that the buyer's optimal strategies are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit of charging the high price is given by $(1-\delta)p_H < p_L$ due to the fact that $\delta > 1 - \frac{p_L}{p_H}$. The low quality seller would find it more profitable to deviate and charge exclusively the low price. A contradiction. We prove sufficiency through claims 5.1-5.2:

Claim 5.1 : *There exists a continuum of pooling equilibrium outcome characterized by* $\phi_H^* = \phi_L^* = 1$ *and* $b_H^*(p_H) = \gamma_H$ *and* $b_L^*(p_H) = 0$ *if* $\delta = \delta_H$, *where* $\gamma_H \geq \left(\frac{1}{1-\delta}\right)\frac{p_L}{p_H}$.

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta = \delta_H$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta = \delta_H$, then $\mu_b^*(H|p_H, H) = A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. By charging the high price, his expected profit is given by $(1-\delta)\gamma_H p_H \geq p_L$. Thus, charging the high price is the best reply for the low quality seller and hence, also for the high quality seller $\delta\gamma_H p_H > p_L$.

Claim 5.2 : *There exists a unique pooling equilibrium outcome characterized by* $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ *and* $b_L^*(p_H) = 0$ *if* $\delta \in \left(\delta_H, 1 - \frac{p_L}{p_H}\right]$.

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \in \left(\delta_H, 1 - \frac{p_L}{p_H}\right]$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_H$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Since the buyer accepts to trade at the high price only if she observes the high signal, his expected profit of charging the high price is given by $(1-\delta)p_H \geq p_L$, charging the high price is the best reply for this seller and consequently, it is so for the high quality seller too. Uniqueness follows trivially.

This completes the proof of lemma 5.

Lemma 6 : *If* $p_H > \bar{v}$ *and* $\delta_H > 1 - \frac{p_L}{p_H}$, *no fraudulent pooling equilibrium exists.*

Proof. Suppose not, so that if $p_H > \bar{v}$ and $\delta_H > 1 - \frac{p_L}{p_H}$, a fraudulent pooling equilibrium exists. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (6). If $\delta < \delta_H$, then $A > \mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L)$ so that the buyer's optimal strategies are $b_H^*(p_H) = b_L^*(p_H) = 0$. Any seller who charged the high price would see his offer rejected and therefore, a low quality seller would be better off by being honest and charging only the low price. If $\delta \geq \delta_H$, $\mu_b^*(H|p_H, H) \geq A > \mu_b^*(H|p_H, L)$, so that the buyer's optimal strategies are $b_H^*(p_H) \leq 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit of charging the high price is at most $(1-\delta)p_H < p_L$ due to the fact that $\delta > 1 - \frac{p_L}{p_H}$. The low quality seller would find it more profitable to deviate and charge exclusively the low price. A contradiction.

Lemma 7 : *If $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$, Hybrid Equilibria exist if and only if $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup \left[1 - \frac{p_L}{p_H}, 1\right)$.*

Proof. For necessity, suppose not, so that if $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$, hybrid equilibria exist if $\delta \in \left[\delta_H, 1 - \frac{p_L}{p_H}\right)$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (6). Since $\delta \geq \delta_H$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$ $\forall \phi_L \in (0, 1)$ so that the buyer's optimal strategies are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. The low quality seller's expected profit of charging the high price is given by $(1-\delta)p_H > p_L$. The low quality seller would find it more profitable to deviate and charge exclusively the high price. A contradiction. We prove sufficiency through claims 7.1-7.2:

Claim 7.1 : *If $p_H > \bar{v}$, Hybrid Equilibria exist if $\delta < \min\{\delta_H, 1 - \frac{p_L}{p_H}\}$.*

Proof. Consider the equilibrium outcome characterized by $\phi_H^* = 1$, $\phi_L^* = \bar{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{\delta}{1-\delta}\right)\left(\frac{v_H - p_H}{p_H - v_L}\right)$, $b_H^*(p_H) = \underline{\gamma_{\bar{H}}}(\delta) \equiv \left(\frac{1}{1-\delta}\right)\frac{p_L}{p_H}$ and $b_L^*(p_H) = 0$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \leq 1 - \frac{p_L}{p_H}$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. In addition, $\mu_b^*(H|p_H, H) = A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Since the buyer accepts to trade at the high price only if she observes the high signal with probability $\underline{\gamma_{\bar{H}}}(\delta)$, the expected profit of charging the high price for the low quality seller is given by $(1-\delta)\underline{\gamma_{\bar{H}}}(\delta)p_H = p_L$. The low quality seller is indifferent between charging both prices and therefore, a randomization strategy is an optimal reply for this seller. Similarly, charging only the high price is the high quality seller's optimal response for the buyer's strategy. Uniqueness follows trivially.

Claim 7.2 : *If $p_H > \bar{v}$, Hybrid Equilibria exist if $\delta \geq 1 - \frac{p_L}{p_H}$.*

Proof. Consider the equilibrium outcome characterized by $\phi_H^* = 1$, $\phi_L^* = \underline{\phi}(\delta)$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = \underline{\gamma_{H\bar{L}}}(\delta)$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \geq 1 - \frac{p_L}{p_H}$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. In addition, $\mu_b^*(H|p_H, H) > A = \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Since the buyer accepts to trade at the high price with probability one if she observes the high signal and with probability strictly less than one if she observes the low signal, the expected profit of charging the high price for the low quality seller is given by $[(1-\delta) + \delta\underline{\gamma_{H\bar{L}}}(\delta)]p_H = p_L$. The low quality seller is indifferent between charging both prices and therefore, a randomization strategy is a best reply for this seller. Similarly, charging only the high price is the high quality seller's optimal response for the buyer's strategy. Uniqueness follows trivially.

This completes the proof of lemma 7.

The proof of proposition 2 is completed. Q.E.D.

Let us define:

$$n_{pH\bar{L}}(\delta,\gamma) \equiv \frac{\beta\delta(1-\gamma)[\delta_h + (1-\delta_h)\gamma]}{\delta(1-\gamma) - (1-\beta\delta_h - \beta(1-\delta_h)\gamma)\left(1-\frac{p_L}{p_H}\right)}$$

$$\bar{n}_p \equiv \frac{\beta\delta\delta_h}{\delta - (1-\beta\delta_h)\left(1-\frac{p_L}{p_H}\right)} \quad ; \quad n_{p\bar{H}}(\delta,\gamma) \equiv \frac{\beta\delta_h\gamma[1-(1-\delta)\gamma]}{\frac{p_L}{p_H} + \beta\delta_h\gamma\left(1-\frac{p_L}{p_H}\right) - (1-\delta)\gamma}$$

$$\underline{n} \equiv \bar{n}_p - \frac{\beta(1-\pi)\delta(1-\underline{\phi}(\delta))}{\delta - (1-\beta\delta_h)\left(1-\frac{p_L}{p_H}\right)}\left(1-\frac{p_L}{p_H} - \delta\right)$$

$$\bar{n}_h \equiv \bar{n}_p - \frac{\beta(1-\pi)\delta(1-\bar{\phi}(\delta))}{\delta - (1-\beta\delta_h)\left(1-\frac{p_L}{p_H}\right)}\left(1 - \frac{p_L}{p_H} - \delta\right).$$

$$\overline{\gamma_{H\bar{L}}}(\delta) \equiv \frac{\delta - (1-\beta\delta_h)\left(1-\frac{p_L}{p_H}\right)}{\delta - \beta(1-\delta_h)\left(1-\frac{p_L}{p_H}\right)} \text{[20]} \quad ; \quad \bar{\gamma}_{\bar{H}}(\delta) \equiv \frac{\frac{p_L}{p_H}}{1-\delta-\beta\delta_h\left(1-\frac{p_L}{p_H}\right)} \text{[21]}$$

$$n_{hH\bar{L}}(\delta,\gamma) \equiv \frac{\beta\delta(1-\gamma)\left\{[\delta_h + (1-\delta_h)\gamma] - (1-\pi)(1-\underline{\phi}(\delta))\left(1-\frac{p_L}{p_H} - \delta(1-\gamma)\right)\right\}}{\delta(1-\gamma) - [1-\beta\delta_h - \beta(1-\delta_h)\gamma]\left(1-\frac{p_L}{p_H}\right)} \text{[22]}$$

$$n_{h\bar{H}}(\delta,\gamma) \equiv \frac{\beta[1-(1-\delta)\gamma]\left\{[\pi\delta + (1-\pi)(1-\delta)\bar{\phi}]\gamma + (1-\pi)(1-\bar{\phi})\frac{p_L}{p_H}\right\}}{\frac{p_L}{p_H} + \beta\delta_h\gamma\left(1-\frac{p_L}{p_H}\right) - (1-\delta)\gamma}. \text{[23]}$$

Notice that for $\delta > \delta_s$, $\delta > (1-\beta\delta_h)\left(1-\frac{p_L}{p_H}\right)$, and $\bar{n}_p$, $\underline{n}$, $\bar{n}_h$ are positive, decreasing and convex in δ. Furthermore, if $\delta_s < \delta < \delta_H$, then $\bar{\phi}(\delta) < 1$ and $\bar{n}_p > \bar{n}_h$. If $\delta > \max\{\delta_H, \delta_s\}$, then $\bar{\phi}(\delta) > 1$ and $\bar{n}_p < \bar{n}_h$. As $\delta \to \delta_s$, $\bar{n}_p, \underline{n}, \bar{n}_h \to +\infty$; as $\delta \to 1 - \frac{p_L}{p_H}$, $\bar{n}_p, \underline{n}, \bar{n}_h \to 1$.

Proof of Proposition 5. We prove this proposition through a series of lemmas.

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[20]It is increasing and concave in δ for $\delta > \delta_s$. As $\delta \to \delta_s$, $\overline{\gamma_{H\bar{L}}} \to 0$; $\delta \to 1$, $\overline{\gamma_{H\bar{L}}} \to \frac{1-(1-\beta\pi)\left(1-\frac{p_L}{p_H}\right)}{1-\beta(1-\pi)\left(1-\frac{p_L}{p_H}\right)} \equiv \bar{\gamma}$

[21]It is increasing and convex in δ for $\delta < \delta_t$. As $\delta \to \frac{1}{2}$, $\bar{\gamma}_{\bar{H}} \to \frac{2\frac{p_L}{p_H}}{1-\beta\left(1-\frac{p_L}{p_H}\right)}$; as $\delta \to \delta_s$, $\bar{\gamma}_{\bar{H}} \to 1$.

[22]It is decreasing in δ, increasing in γ for $\delta > \delta_s$ and decreasing in π for $\gamma < \overline{\gamma_{H\bar{L}}}$. Furthermore, it is convex in δ for $\gamma < \bar{\gamma}$. As $\gamma \to \underline{\gamma_{H\bar{L}}}(\delta)$, $n_{hH\bar{L}}(\delta,\gamma) \to 1$; as $\gamma \to 0$, $n_{hH\bar{L}}(\delta,\gamma) \to \underline{n}$; $\gamma \to \overline{\gamma_{H\bar{L}}}(\delta)$, $n_{hH\bar{L}}(\delta,\gamma) \to +\infty$ for $\delta > \delta_s$; as $\delta \to \delta_s$, $n_{hH\bar{L}}(\delta,\gamma) \to +\infty$.

[23]It is increasing in π and increasing in γ for $\delta \leq \delta_u$. For $\delta \in (\delta_u, \delta_t)$, it is increasing in γ if and only if $\gamma < \frac{D\frac{p_L}{p_H} - E}{D\left[1-\delta-\beta\delta_h\left(1-\frac{p_L}{p_H}\right)\right]} \equiv \gamma_{h\bar{H}1}$ where $D \equiv 2(1-\delta)[\pi\delta + (1-\pi)(1-\delta)\bar{\phi}]$ and $E \equiv \left(2D\delta_h\frac{p_L}{p_H}\left(1-\frac{p_L}{p_H}\right)[-(1-\delta)+\beta\delta_h]\left[1-\beta(1-\pi)(1-\bar{\phi})\left(1-\frac{p_L}{p_H}\right)\right]\right)^{-\frac{1}{2}}$. For $\delta > \delta_t$, it is increasing in γ if and only if $\gamma < \frac{D\frac{p_L}{p_H} + E}{D\left[1-\delta-\beta\delta_h\left(1-\frac{p_L}{p_H}\right)\right]} \equiv \gamma_{h\bar{H}2}$. As $\gamma \to \underline{\gamma_{\bar{H}}}(\delta)$, $n_{h\bar{H}}(\delta,\gamma) \to 1$; as $\gamma \to \bar{\gamma}_{\bar{H}}(\delta)$, $n_{h\bar{H}}(\delta,\gamma) \to +\infty$ for $\delta \leq \delta_s$; as $\gamma \to 1$, $n_{h\bar{H}}(\delta,\gamma) \to \bar{n}_h$, for $\delta > \delta_s$.

Lemma 8: *If $p_H < \bar{v}$, Fraudulent Pooling Equilibria exist for all possible levels of competition in the market if $\delta \leq \max\{\delta_s, \delta_L\}$.*

Claim 8.1 : *If $p_H < \bar{v}$, and for all market competition levels, there exists a unique pooling equilibrium outcome characterized by the following strategies $\phi_H^* = \phi_L^* = 1$ and $b_H^*(p_H) = b_L^*(p_H) = 1$ if $\delta < \delta_L$.*

Proof. The proof follows by the same reasonings used in the proof of Claim 2.1. Q.E.D.

Claim 8.2 : *If $p_H < \bar{v}$, and for all market competition levels, there exists a continuum of pooling equilibrium outcomes characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = \gamma$ if $\delta = \delta_L$, where $\gamma \in [0, 1]$ if $\delta_L \leq \delta_s$ and $\gamma \in [\overline{\gamma_{H\bar{L}}}(\delta), 1]$ otherwise.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta = \delta_L$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta = \delta_L$, then $\mu_b^*(H|p_H, H) > A$ and $\mu_b^*(H|p_H, L) = A$. By (2), the buyer's strategy is rational with respect to them. Now consider the low and high quality sellers respectively. By charging the high price, their expected lifetime utilities are given by:

$$V_L(p_H) = [(1-\delta) + \delta\gamma](p_H + \beta V) + \frac{\beta}{n}\delta(1-\gamma)V + \beta\left(1 - \frac{1}{n}\right)\delta(1-\gamma)V_u$$

$$V_H(p_H) = [\delta + (1-\delta)\gamma](p_H + \beta V) + \frac{\beta}{n}(1-\delta)(1-\gamma)V + \beta\left(1 - \frac{1}{n}\right)(1-\delta)(1-\gamma)V_u$$

In equilibrium, $V = \pi V_H(p_H) + (1-\pi)V_L(p_H)$, which can be written as:

$$V = [\delta_h + (1-\delta_h)\gamma](p_H + \beta V) + \frac{\beta}{n}(1-\delta_h)(1-\gamma)V + \beta\left(1 - \frac{1}{n}\right)(1-\delta_h)(1-\gamma)V_u$$

and

$$V_u = [\delta_h + (1-\delta_h)\gamma](0 + \beta V_u) + \frac{\beta}{n}(1-\delta_h)(1-\gamma)V + \beta\left(1 - \frac{1}{n}\right)(1-\delta_h)(1-\gamma)V_u$$

so that

$$(V - V_u)_{pH\bar{L}} = \frac{[\delta_h + (1-\delta_h)\gamma]p_H}{1 - \beta[\delta_h + (1-\delta_h)\gamma]} \tag{15}$$

In equilibrium, a one shot deviation must be not profitable for none of the sellers. Since $V_H(p_H) >$

$V_L(p_H)$, this dishonesty condition can be written as,

$$[(1-\delta)+\delta\gamma]p_H - p_L \geq \beta\left(1-\frac{1}{n}\right)\delta(1-\gamma)(V-V_u)_{pH\bar{L}} \tag{16}$$

If we substitute (15) into (16), condition (16) can be shown to be satisfied for all market competition levels if $\delta_L \leq \delta_s$ & $\gamma \in (0,1)$ or if $\delta_L > \delta_s$ & $\gamma \geq \overline{\gamma_{H\bar{L}}}(\delta)$.

Claim 8.3 : *If $p_H < \bar{v}$ and for all market competition levels, there exists a unique pooling equilibrium outcome characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$ if $\delta \in (\delta_L, \delta_s]$.*

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \in \left(\delta_L, 1-\frac{p_L}{p_H}\right]$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Consider now the sellers. By following a similar analysis as the one used in the proof of Claim 8.2, we arrive at the following conditions:

$$(V-V_u)_{pH} = \frac{\delta_h p_H}{1-\beta\delta_h} \tag{17}$$

In equilibrium, a one shot deviation must be not profitable for none of the sellers. Since $V_H(p_H) > V_L(p_H)$, this dishonesty condition can be written as,

$$(1-\delta)p_H - p_L \geq \beta\left(1-\frac{1}{n}\right)\delta(V-V_u)_{pH} \tag{18}$$

If we substitute (17) into (18), condition (18) can be shown to be satisfied for all market competition levels if $\delta_L < \delta \geq \delta_s$. Uniqueness follows trivially.

This completes the proof of lemma 8.

Lemma 9 : *If $p_H \leq \bar{v}$ and for $n \leq \bar{n}_p$, there exists a unique fraudulent pooling equilibrium outcome characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$ if $\delta \in \left(\max\{\delta_s, \delta_L\}, 1-\frac{p_L}{p_H}\right)$.*

Proof. The proof follows by using an argument similar as the one used in the proof of claim 8.3.

Lemma 10 : *If $p_H = \bar{v}$ and for all market competition levels, there exists a unique fraudulent pooling equilibrium outcome characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$ if $\delta \leq \delta_s$*

If $p_H = \bar{v}$ and for all market competition levels, there exists a unique fraudulent pooling equilibrium outcome characterized by $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$ if $\delta \in \left(\delta_s, 1 - \frac{p_L}{p_H}\right)$.

Proof. If $p_H = \bar{v}$, then $\pi = A$ and $\delta_L = \frac{1}{2}$. The proof follows by using an argument similar as the one used in the proof of claim 8.3 .

Lemma 11 : *If $p_H \leq \bar{v}$ no fraudulent pooling equilibrium exists for any other possible combination of δ and n.*

Claim 11.1 : *If $p_H \leq \bar{v}$, and for $n \geq \bar{n}_p$, no fraudulent pooling equilibrium exists if $\delta \in \left(\max\{\delta_s, \delta_L\}, 1 - \frac{p_L}{p_H}\right)$.*

Proof. Suppose not, so that if $p_H \leq \bar{v}$, and for $n \geq \bar{n}_p$, pooling equilibria exist if $\delta \in \left(\max\{\delta_s, \delta_L\}, 1 - \frac{p_L}{p_H}\right)$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (6). Since $\delta \geq \delta_L$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$ $\forall \phi_L \in (0, 1)$ so that the buyer's optimal strategies are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. Condition (18) is violated. The low quality seller would find it more profitable to deviate and charge exclusively the low price. A contradiction.

Claim 11.2 : *If $p_H \leq \bar{v}$, and for $\forall n$, no fraudulent pooling equilibrium exists if $\delta > \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$ or if $\delta = 1 - \frac{p_L}{p_H} > \delta_L$.*

Proof. Suppose not, so that if $p_H \leq \bar{v}$, and for $n \geq \bar{n}_p$, pooling equilibria exist if $\delta > \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$ or if $\delta = 1 - \frac{p_L}{p_H} > \delta_L$. By Bayes consistency, the posterior beliefs $\mu_b^*(H|p_H, s)$ must be given by equation (6). Since $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$ so that the buyer's optimal strategies are $b_H^*(p_H) = 1$ and $b_L^*(p_H) = 0$. If $\delta \geq 1 - \frac{p_L}{p_H}$, then the low quality seller would find it more profitable to deviate and charge exclusively the low price, since $(1 - \delta)p_H < p_L$ and the seller is guaranteed to remain matched to the buyer in the following period. A contradiction. This completes the proof of lemma 11.

Lemma 12 : *If $p_H \leq \bar{v}$, and for all possible levels of competition in the market, Hybrid Equilibria exist if $\delta \in \left(\max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}, 1\right)$.*

Proof. Consider a continuum of hybrid equilibrium outcome characterized by $\phi_H^* = 1$, $\phi_L^* = \underline{\phi}(\delta) \equiv \left(\frac{\pi}{1-\pi}\right)\left(\frac{1-\delta}{\delta}\right)\left(\frac{v_H - p_H}{p_H - v_L}\right)$, $b_H^*(p_H) = 1$ and $b_L^*(p_H) = \gamma \in (\underline{\gamma_{H\bar{L}}}(\delta), \overline{\gamma_{H\bar{L}}}(\delta))$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta > \max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_L$, then $\mu_b^*(H|p_H, H) > A = \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the sellers. It can be shown that in equilibrium,

$$(V - V_u)_{hH\bar{L}} = \frac{(\delta_h(\underline{\phi}(\delta)) + \delta_l(\underline{\phi}(\delta))\gamma)p_H + (1-\pi)(1-\underline{\phi}(\delta))p_L}{1 - \beta(\delta_h(\underline{\phi}(\delta)) + \delta_l(\underline{\phi}(\delta))\gamma + (1-\pi)(1-\phi))} \tag{19}$$

In equilibrium, the low quality seller must be indifferent between being honest or dishonest. This condition is given by equation (11). If we substitute (19) into (11), condition (11) can be shown to be satisfied for $n = n_{hH\bar{L}}(\delta, \gamma)$ if $\delta \in \left(\max\left\{\delta_L, 1 - \frac{p_L}{p_H}\right\}, 1\right)$ and $\gamma \in (\underline{\gamma_{H\bar{L}}}(\delta), \overline{\gamma_{H\bar{L}}}(\delta))$. Since $n_{hH\bar{L}}(\delta, \gamma)$ is increasing in γ and as $\gamma \to \underline{\gamma_{H\bar{L}}}(\delta)$, $n_{hH\bar{L}}(\delta, \gamma) \to 1$ and as $\gamma \to \overline{\gamma_{H\bar{L}}}(\delta)$, $n_{hH\bar{L}}(\delta, \gamma) \to +\infty$ for $\delta > \delta_s$, then this equilibrium exists for all market competition levels.

Lemma 13 : *If $p_H \leq \bar{v}$ and $n \geq \underline{n}$, hybrid equilibria exist if $\delta \in \left(\max\{\delta_s, \delta_L\}, 1 - \frac{p_L}{p_H}\right]$.*

Proof. The proof follows by using an argument similar as the one used in the proof of lemma 12. If we substitute (19) into (11), condition (11) can be shown to be satisfied for $n = n_{hH\bar{L}}(\delta, \gamma)$ if $\delta \in \left(\max\{\delta_s, \delta_L\}, 1 - \frac{p_L}{p_H}\right]$ and $\gamma < \overline{\gamma_{H\bar{L}}}(\delta)$. Since $n_{hH\bar{L}}(\delta, \gamma)$ is increasing in γ and as $\gamma \to 0$, $n_{hH\bar{L}}(\delta, \gamma) \to \underline{n}$ and as $\gamma \to \overline{\gamma_{H\bar{L}}}(\delta)$, $n_{hH\bar{L}}(\delta, \gamma) \to +\infty$ for $\delta > \delta_s$, then this equilibrium exists for all market competition levels such that $n \geq \underline{n}$.

Lemma 14 : *If $p_H \leq \bar{v}$, no hybrid equilibrium exists for any other possible combination of δ and n.*

Proof. Suppose not, so that a hybrid equilibrium exists either if $\delta \leq \max\{\delta_s, \delta_L\}$, or if $\delta \in \left(\max\{\delta_s, \delta_L\}, 1 - \frac{p_L}{p_H}\right]$ & $n \leq \underline{n}$. First, suppose that $\delta < \delta_L$. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s, \phi)$ given by equation (6). These beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta \leq \delta_L$, then $\mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L) > A$ $\forall \phi < 1$. By (2), the buyer's optimal strategy is to always accept both price offers. Now consider the low and high quality sellers. By charging only the high price instead of the low price, they obtain a relatively higher current profit and they are guaranteed to remain matched with the buyer in the next period. Thus randomization is not an optimal strategy for the seller. Secondly, suppose that $\delta \in (\delta_L, \delta_s]$. Then, $\mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L) > A \ \forall \phi < \underline{\phi}(\delta)$ but then the above reasoning applies; $\mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L) = A$ if $\phi = \underline{\phi}(\delta)$. The buyer's strategy is rational if and only if the high signal customer always accepts the high price offer while the low signal customer accepts the high price offer with probability $\gamma \in [0, 1]$. Consider now the low quality seller. By using an argument similar as the one used in the proof of lemma 12, condition (11) can be shown to be violated if $\delta \leq \delta_s$. The low quality seller has incentives to deviate and strictly outright cheat. $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$ if $\underline{\phi}(\delta) < \phi < 1$. The buyer's strategy is rational if and only if the high signal customer always accepts the high price offer while the low signal customer always rejects the high price offer. Consider now the low quality seller. It can be shown that in equilibrium, condition (9) holds. Furthermore, the low quality seller must be indifferent between being honest or dishonest. This condition can be written as,

$$(1 - \delta)p_H - p_L = \beta\delta \left(1 - \frac{1}{n}\right)(V - V_u)_{hH} \tag{20}$$

Condition (20) can be shown to be violated if $\delta \leq \delta_s$. The low quality seller has incentives to deviate and strictly outright cheat. Thirdly, suppose that $\delta \in \left(\max\{\delta_s, \delta_L\}, 1 - \frac{p_L}{p_H}\right]$. Then, $\mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L) > A \ \forall \phi < \underline{\phi}(\delta)$ but then the same reasoning provided above applies; $\mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L) = A$ if $\phi = \underline{\phi}(\delta)$. The buyer's strategy is rational if and only if the high signal customer always accepts the high price offer while the low signal customer accepts the high price offer with probability $\gamma \in [0, 1]$. Consider now the low quality seller. By using an argument similar as the one used in the proof of lemma 12, condition (11) can be shown to be violated if $n \leq \underline{n}$. The low quality seller has incentives to deviate and strictly outright cheat; $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$ if $\underline{\phi}(\delta) < \phi < 1$. The buyer's strategy is rational if and only if the high signal customer always accepts the high price offer while the low signal customer always rejects the high price offer. Consider now the low quality seller. Condition (20) can be shown to be violated if $n \leq \underline{n}$. The low quality seller has incentives to deviate and strictly outright cheat.

This completes the proof of proposition 5. Q.E.D.

Proof of Proposition 6. We prove this proposition through a series of lemmas.

Lemma 15: *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *Fraudulent Pooling Equilibria exist for all possible competition levels in the market if* $\delta \in [\delta_H, \delta_s]$.

Proof.

Claim 15.1 : *For all possible competition levels in the market, there exists a continuum of pooling equilibrium outcome characterized by* $\phi_H^* = \phi_L^* = 1$ *and* $b_H^*(p_H) = \gamma_H$ *and* $b_L^*(p_H) = 0$ *if* $\delta = \delta_H \leq \delta_s$, *where* $\gamma_H \geq \bar{\gamma_{\bar{H}}}(\delta)$.

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta = \delta_H$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta = \delta_H$, then $\mu_b^*(H|p_H, H) = A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. In equilibrium,

$$(V - V_u)_{p\bar{H}} = \frac{\delta_h \gamma p_H}{1 - \beta \delta_h \gamma} \tag{21}$$

In equilibrium, a one shot deviation must be not profitable for none of the sellers. Since $V_H(p_H) > V_L(p_H)$, this dishonesty condition can be written as,

$$(1 - \delta)\gamma p_H - p_L \geq \beta \left(1 - \frac{1}{n}\right) [1 - (1 - \delta)\gamma](V - V_u)_{p\bar{H}} \tag{22}$$

If we substitute (21) into (22), condition (22) can be shown to be satisfied for all market competition levels if $\delta = \delta_H \leq \delta_s$ & $\gamma \geq \bar{\gamma_{\bar{H}}}(\delta)$.

Claim 15.2 : *For all possible competition levels in the market, there exists a unique pooling equilibrium outcome characterized by* $\phi_H^* = \phi_L^* = 1$, $b_H^*(p_H) = 1$ *and* $b_L^*(p_H) = 0$ *if* $\delta \in (\delta_H, \delta_s]$.

Proof. Consider the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \in (\delta_H, \delta_s]$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta > \delta_H$, then $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. Following a similar reasoning as in the proof of Claim 8.3, if we substitute (17) into (18), condition (18) can be shown to be satisfied for all market competition levels if $\delta_H < \delta \leq \delta_s$. Uniqueness follows trivially.

This completes the proof of lemma 15.

Lemma 16: *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *In less competitive markets, defined as* $n \leq \bar{n}_p$, *fraudulent pooling equilibria exist if* $\delta \in \Big(\max\{\delta_H, \delta_s\}, 1 - \frac{p_L}{p_H}\Big)$.

Proof. The proof follows by using an argument similar as the one used in the proof of claim 15.2.

Lemma 17: *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *If* $\delta = \delta_H > \delta_s$, *then fraudulent pooling equilibria also exist if* $n \leq n_{p\bar{H}}(\delta, \gamma)$.

Proof. The proof follows by using an argument similar as the one used in the proof of claim 15.1. Condition (22) can be shown to be satisfied for $n \leq n_{p\bar{H}}(\delta, \gamma)$ if $\delta = \delta_H > \delta_s$ & $\gamma \geq \underline{\gamma_{\bar{H}}}(\delta)$

Lemma 18: *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *No fraudulent pooling equilibrium exists for any other possible combination of* δ *and* n

Claim 18.1 : *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *If* $\delta = \delta_H > \delta_s$, *then no fraudulent pooling equilibria exist for* $n > n_{p\bar{H}}(\delta, \gamma)$.

Proof. Suppose not. By using an argument similar as the one used in the proof of lemma 17, condition (22) can be shown to be violate for $n > n_{p\bar{H}}(\delta, \gamma)$ if $\delta = \delta_H > \delta_s$. The low quality seller strictly prefers to be totally honest.

Claim 18.2: *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *If* $\delta \in \Big(\max\{\delta_H, \delta_s\}, 1 - \frac{p_L}{p_H}\Big)$, *then no fraudulent pooling equilibria exist for* $n > \bar{n}_p$.

Proof. The proof follows by using an argument similar as the one used in the proof of lemma 16. Condition (18) can be shown to be violated for $n > \bar{n}_p$ if $\delta \in \Big(\max\{\delta_H, \delta_s\}, 1 - \frac{p_L}{p_H}\Big)$. The low quality seller strictly prefers to be totally honest.

Claim 18.3: *Suppose* $p_H > \bar{v}$ *and* $\delta_H \leq 1 - \frac{p_L}{p_H}$. *If* $\delta \geq 1 - \frac{p_L}{p_H} > \delta_H$, *then no fraudulent pooling equilibria exist for any market competition level.*

Proof. The proof follows by using an argument similar as the one used in the proof of claim 11.2.

This completes the proof of lemma 18.

Lemma 19 : *Suppose that $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$. For all market competition levels, Hybrid Equilibria exist if $\delta \in \left(\frac{1}{2}, \delta_H\right) \cup \left(1 - \frac{p_L}{p_H}, 1\right)$.*

Proof. Consider the equilibrium outcome characterized by $\phi_H^* = 1$, $\phi_L^* = \bar{\phi}(\delta) \equiv$, $b_H^*(p_H) \in (\underline{\gamma_{\bar{H}}}(\delta), \max\{\bar{\gamma_{\bar{H}}}(\delta), 1\})$ and $b_L^*(p_H) = 0$ and the buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ given by equation (6). To verify that (μ^*, ϕ^*, b^*) if $\delta \leq 1 - \frac{p_L}{p_H}$ is a SSPBE, notice that these beliefs are by construction consistent with Bayes' rule along the equilibrium path. In addition, $\mu_b^*(H|p_H, H) = A > \mu_b^*(H|p_H, L)$. By (2), the buyer's strategy is rational with respect to them. Now consider the low quality seller. In equilibrium, the low quality seller must be indifferent between being honest or dishonest. This condition is given by equation (13). If we substitute (14) evaluated at $\bar{\phi}(\delta)$ into (13), condition (13) can be shown to be satisfied for $n = n_{h\bar{H}}(\delta, \gamma)$ if $\delta \leq \delta_s$ and $\gamma \in (\underline{\gamma_{\bar{H}}}(\delta), \bar{\gamma_{\bar{H}}}(\delta))$, or if $\delta \in (\delta_s, \delta_H)$ and $\gamma > \underline{\gamma_{\bar{H}}}(\delta)$. Since $n_{h\bar{H}}(\delta, \gamma)$ is increasing in γ if $\delta \leq \delta_s$ and as $\gamma \to \underline{\gamma_{\bar{H}}}(\delta)$, $n_{h\bar{H}}(\delta, \gamma) \to 1$ and as $\gamma \to \bar{\gamma_{\bar{H}}}(\delta)$, $n_{h\bar{H}}(\delta, \gamma) \to +\infty$ if $\delta \leq \delta_s$, then this equilibrium exists for all market competition levels if $\delta \leq \delta_s$. Notice that $\gamma \to 1$, $n_{h\bar{H}}(\delta, \gamma) \to \bar{n}_h$. Since $n_{h\bar{H}}(\delta, \gamma)$ is increasing in γ if $\delta \leq \delta_u$, and it is increasing in γ at first and then it may decrease in γ if $\delta \leq \delta_u$, then this equilibrium exists for $n \geq \bar{n}_h$. By an argument similar as the one used in the proof of lemma 13, a hybrid equilibrium exists if $\delta \in (\delta_s, \delta_H)$ for $n \leq \underline{n}$, where $\underline{n} < \bar{n}_h$ if $\delta \in \left(\delta_s, 1 - \frac{p_L}{p_H}\right)$. By an argument similar as the one used in the proof of lemma 12, a hybrid equilibrium exists if $\delta \in \left(1 - \frac{p_L}{p_H}, 1\right)$

Lemma 20 : *Suppose that $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$. Hybrid Equilibria exist for $n \geq \underline{n}$, if $\delta \in \left(\max\{\delta_H, \delta_s\}, 1 - \frac{p_L}{p_H}\right)$.*

Proof. The proof follows by using an argument similar as the one used in the proof of lemma 13.

Lemma 21 : *Suppose that $p_H > \bar{v}$ and $\delta_H \leq 1 - \frac{p_L}{p_H}$. No hybrid equilibrium exists for any other possible combination of δ and n.*

Proof. Suppose not so that hybrid equilibria exist for $n < \underline{n}$, if $\delta \in \left(\max\{\delta_H, \delta_s\}, 1 - \frac{p_L}{p_H}\right)$. We arrive at a contradiction by using an argument similar as the one of the arguments used in the proof of lemma 14.

Lemma 22 : *Suppose that $\delta_H > 1 - \frac{p_L}{p_H}$. No fraudulent pooling equilibrium exists and hybrid equilibria exist $\forall\delta$ & $\forall n$.*

Proof. Suppose not so that pooling equilibria exist for some δ. The buyer's posterior beliefs $\mu_b^*(H|p_H, s)$ are given by equation (6). These beliefs are by construction consistent with Bayes' rule along the equilibrium path. If $\delta < \delta_H$, they satisfy $A > \mu_b^*(H|p_H, H) > \mu_b^*(H|p_H, L)$. By (2), the buyer's optimal strategy is to reject the high price offer independently of the signal realization. But then, any seller would have incentives to deviate and be honest. If $\delta > \delta_H$, then they satisfy $\mu_b^*(H|p_H, H) > A > \mu_b^*(H|p_H, L)$. By (2), the buyer's optimal strategy is to accept the high price offer if and only if the high signal is realized. But then, any seller would have incentives to deviate and be honest since both the current profit would be greater and the seller is guaranteed to be matched with the buyer in the following period. A contradiction. The proof that hybrid equilibria exist $\forall\delta$ & $\forall n$ follows by using an argument similar as the one used in the proof of lemma 19. The proof of proposition 6 is completed. Q.E.D.

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