Two-Sided Search in Credit Markets with Imperfect Information

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Abstract

This paper presents a two-sided search model of bank loan markets with imperfect information. It is shown that the average rate of loan repayments by a firm equipped with a search technology for investment selection decreases following a rise in interest rate due to the unobservability of project types by any banks. Under uncertainty on borrower types, the bank incorporates firms’ repayment policy as a Stackelberg reaction function in its search model, which can be used to integrate its selection of customers with its choice of interest rates and to partition different types of firms into lending groups. Risky (or safe) groups are thus made to pay a high (or low) interest rate so that moral hazard is reduced significantly. It is demonstrated that there is neither adverse selection nor credit rationing at Bertrand competitive rates in search equilibrium.

Keywords: Bank loan, Default risk, Uncertainty, Two-sided search.

JEL Classification: C73, D83, G21.

1 Introduction

Most of traditional studies in the credit literature focus on the adverse selection and incentive effects in bank loan markets with the asymmetry of information involving borrowers’ hidden characteristics and/or hidden actions. This literature has developed largely on the basis of Stiglitz and Weiss’s (1981) work by justifying credit rationing as the equilibrium excess demand for credit, and Jaffee and Stiglitz (1990) provide an extensive survey of the literature. Our paper is motivated by another type of credit market imperfection, which is intertemporal uncertainty on investment.
outcomes faced by a firm and on lending prospects faced by a bank. This “cross-time” uncertainty is to be addressed together with the "cross-sectional" asymmetry of information in a two-sided search framework, in which the firm that borrows for production is looking for acceptable projects to invest while the bank is seeking acceptable customers for lending.

There are a number of papers that questioned the credit rationing equilibrium. By employing various instruments such as equity financing (see Cho (1986)), variable loan sizes (Grinblatt and Hwang (1989)), collateral requirements (Bester (1985)), co-investment and capital structure (Leland and Pyle (1977) and Brennan and Kraus (1987)), banks can screen loan applicants or induce their self-selection, and hence there cannot be any credit rationing. However, it may be too costly for a bank to resolve informational asymmetry in complex real situations by utilizing the above instruments. As a matter of fact, credit rationing still exists when the asymmetry of information is so severe that the bank cannot fully reveal the risk types of firms or projects. Hellmann and Stiglitz (2000) prove that there may actually be credit rationing or/and equity rationing if heterogeneous investors (debt vs. equity) compete to finance the projects of firms that have substantial private information. Based on the analysis of a Bayesian bank with a simple screening technology, Amano (1999) shows that a trade-off between information-gathering costs and higher profits obtained from identifying risky applicants can be used by the bank to derive who are to be rationed credit and by how much.

As underlined by Greenwald and Stiglitz (1987), there is a similarity between credit and labor markets in terms of informational asymmetry. It is largely in static settings, however, in which both these markets are studied in a unified way, and uncertainty has not been well addressed through the use of dynamic framework in the credit literature. The models on the application of search theory to credit issues are few, for example. In the spirit of Diamond (1990), Wasmer and Weil (2001) take a leaf out of the macro-labor literature to build a search model of the interaction between credit and labor markets, which are treated by symmetry. Given that credit and jobs are rationed due to search frictions, they investigate how equilibrium outcomes under economic opportunity uncertainty are affected by sequential pairwise bargaining (first banks vs. firms, then firms vs. workers) in the credit and labor matching processes. These rare search-style models place emphasis on implications for monetary policy or unemployment. The driving force for our search-theoretic study is not on the macroeconomic side but rather from an intention to ascertain the micro-behavior of a bank and a firm, both of which are ex ante imperfectly aware of future risks and opportunities in investment and lending, and will still face ex post asymmetric information after the realization of investment uncertainty.

The two-sided search model in this paper is developed to explore strategic interactions between banks and firms in their forward-looking policy-making processes. The firm’s project-search model is nested in the bank’s customer-search model in the context of a sequential Stackelberg game of price leadership, with the bank treated
as a price leader and the firm as a price follower. Interest rate as a "price" in the
game is a transmission mechanism that links all the decisions together of both parties
in the market. Firms employ a search technology to optimize their project selection
by taking the interest rate as predetermined by the bank. The influences that the
bank’s choice of interest rate exerts on firms’ investment action are summarized by
their repayment policy as a reaction function. The bank must incorporate in advance
these reaction functions into its policy making, and can also utilize a search technol-
ogy to optimally combine its interest-setting policy closely with its customer-selection
decision.

Both the *ex ante* uncertainty and *ex post* asymmetry of information we want to
deal with are embedded in the search models of both sides of the market. As usually
assumed in the literature, a firm observes the *ex post* realized type of a project
once encountering it, but a bank does not when deciding to finance it, a case of
informational asymmetry which is the ultimate source of moral hazard. What type
of project will arrive in the future is unknown *ex ante* to both parties, a case of
informational symmetry but with uncertainty which affects the firm’s search activity.
The potential arrival of projects carries to the firm an investment opportunity along
with some bankruptcy risk if a project is to be undertaken. Although the quality type
of a firm may be observed *ex post* by the bank after receiving its loan application,
there still remains *ex ante* uncertainty about what types of potential borrowers will
arrive, which potentially brings about adverse selection. Therefore, the bank must
base its search behavior on this lending uncertainty as well as on firms’ reaction
functions which already incorporate project uncertainty in their repayment policy.
By distinguishing between firm type and project type while admitting their linkage
under the above information structure, adverse selection effects are separated from
moral hazard problems so that the two-sided search can display a clear pattern of the
strategic interaction between banks and firms.

The equilibrium rate of interest in this paper is determined by the strategic in-
teraction among banks in a simultaneous-move price-setting game. There may be
different kinds of banks in the market, and several identical banks of each kind.
Banks within each category are Bertrand profit-maximizing competitors, faced with
an equal lending opportunity captured by some distribution of potential borrowers
and by the same arrival rate of their loan applications. The Bertrand competition
condition enables us to close the two-sided search model and to attain the zero-profit
market equilibrium. Banks from different categories usually have different lending
opportunities embodied by distinct distributions and arrival rates, and the hetero-
genecy of banks can also be characterized by specific depositing rates that they offer
savers. Then, different banks must have a different policy about customer selection
and interest rate on the basis of different defaulting risks of various firms. Two-sided
search with two-sided heterogeneity can thus lead to credit partnership formation and
sensible market segmentation in an optimal matching process. Though we consider
only firm heterogeneity, this model is readily extended to a two-sided heterogeneity
situation.

Without bank heterogeneity in this framework, we can still achieve some reasonable pattern of market segmentation based on the endogenized interactions between firm groups’ repayment policies and a representative bank’s group-discriminating interest rates. The repeated use of its search technology enables the bank to rationally group its customers given its limited knowledge of their risk characteristics, to charge different groups a different interest rate, and to eliminate the need for rationing credit. Risky borrowers are made to pay high rates they are willing to, so that cross-subsidies from safe firms to risky ones are lessened, and the moral hazard problem is thus largely alleviated. The adverse selection effect is completely wiped out at the Bertrand rates in search equilibrium, in which the bank can make optimal selection of customers while minimizing their moral hazard so that high rates can still be used to subdue default risk.

The rest of this paper is structured as follows. Section 2 presents the firm’s search model to derive its investment action and repayment policy. Section 3 examines the bank’s search behavior and the Bertrand equilibrium. Section 4 analyzes the bank’s repeated use of its search model that provides a grouping mechanism for separating interest rates. Section 5 concludes the paper followed by a technical Appendix.

2 The Firm’s Search for Acceptable Projects

This section discusses the project search policy of a firm, and we make certain assumptions for expository convenience. Both borrowers and lenders are risk neutral. All projects have the same total cost that is greater than the given amount of equity possessed by the firm, which thus need borrow to lunch a project. We assume away raising funds from public debt markets, and there are neither direct bankruptcy costs nor collateral requirements \(^1\). A bank provides loan financing for projects once accepted by the firm, but the old debt borrowed for a successful project has to be repaid before a new one can be serviced to continue the project. Each firm carries out one project each period, the market value of the firm’s equity reduces to zero once its project fails, and so the financing bank must face a default on its loan assets under limited liability enjoyed by borrowers. Loanable funds are supplied at a riskfree (gross) deposit rate \(\rho_o > 1\), which is paid by the bank to savers and unaffected by interest rates the bank charges firms.

2.1 The Firm’s Search Model

The return \(R\) on an investment project, equal to its sales revenue less non-capital costs, has two possible outcomes each period: success to produce \(R(p)\) with proba-

\(^1\)Relaxing this assumption will not alter main results generated, but would substantially have complicated analyses later on.
bility \( p \), or failure to give zero with probability \( p^c \); where \( x \triangleq 1 - x \) for any \( x \). As assumed in Stiglitz and Weiss (1981) and Ghatak (1999), different projects have the same mean return \( \mu_R (p) = pR (p) = R_o \) for all \( p \in [\underline{p}, 1] \) and \( \underline{p} > 0 \). Hence, projects differ only in their riskiness \( \sigma^2_R (p) \), which increases in \( p^c \) since \( \frac{d \sigma^2_R (p)}{dp} = \left( \frac{R_o}{p} \right)^2 > 0 \); and riskier projects if succeeding produce higher returns since \( \frac{d R(p)}{dp^c} = R_o > 0 \).

The success probabilities or quality types \( p \) of projects are assumed to be i.i.d. over time, with distribution \( G(p) \) and density \( g(p) \) defined on support \([\underline{p}, 1]\). This distribution, with mean \( \mu_p = \mu \) and variance \( \sigma^2_p = \sigma^2 \), is firm-specific, and its full form \( G(p | \mu) \) depends on \( \mu \), which is viewed as a firm’s risk type and suppressed now and then for simplicity. The lower bound of \( G \)’s support is also parameterized as \( \underline{p} = \underline{p}(\mu) \), and \([\underline{p}(\mu), 1]\) defines the project population of firm \( \mu \). The firm observes the type \( p \) (ex post realization) of a project once encountering it while a bank knows nothing but its distribution \( G(p) \) at the time of signing a loan contract to finance it, in which case information is asymmetric between both parties. What type \( p \) of project will arrive in the future is unknown ex ante, and only \( G(p) \) is the current common knowledge, in which case information is symmetric though uncertainty remains with both parties.

Each firm is assumed to borrow one unit of bank loan in order to undertake a \( p \)-type project. The random profit \( \pi_f \) earned each period is given by \( R(p) - r \) with probability \( p \) and zero with probability \( p^c \), where \( r \) is the (gross) interest rate paid to the bank by the firm when successful. The mean profit, \( \pi_e (p) = \mu_{\pi_f} (p) = R_o - pr \), is increasing in default risk \( p^c \) since \( \frac{d \pi_e (p)}{dp^c} = r > 0 \). So, on average, riskier projects are more profitable to the firm.

Consider a forward-looking firm with a \( p \)-type project in hand this period. As a Stackelberg price follower, the firm takes \( r \) as given and faces a choice of the acceptance or rejection of this project while having options to search for other investment opportunities in the subsequent periods. This firm’s search value \( V_f(p) \) is defined by a search model \( V_f(p) = \max \{ \Pi_f(p), U_f \} \), where \( \Pi_f(p) \) is acceptance value and \( U_f \) rejection (or waiting) value.

The acceptance value \( \Pi_f(p) \) to the firm, which chooses to accept project \( p \) encountered this period and behaves optimally in the subsequent periods, is determined by a recursive equation:

\[
\Pi_f(p) = p \left\{ [R(p) - r] + \beta \Pi_f(p) \right\} + p^c \left\{ 0 + \beta U_f \right\},
\]

(1)

where \( \beta \) is a time discount factor. If the accepted project \( p \) is a success to produce profits \([R(p) - r] \) with the debt repaid this period, it will be continued in the next period since the bank is willing to provide refinancing, from which onward the process of generating \( \Pi_f(p) \) repeats itself. Otherwise, the firm failing in the project earns zero, defaults on its debt this period, and will have to return to the market to receive
a waiting value $U_f$ from the next period on. Then, rearranging (1) yields

$$\Pi_f(p) = \frac{\pi_e(p) + \beta \rho^c U_f}{1 - \beta p},$$

which is decreasing in $p$, as shown in appendix (A1). The firm tends to go risky in accepting projects since $\Pi'_f(p^c) > 0$.

The rejection value $U_f$ goes to the firm that chooses to reject project $p$ encountered this period in order to wait for possibly better projects. If the firm obtains zero this period in waiting, and is assumed\(^2\) to encounter a new project of type $p$ in the next period to earn a search value $V_f(p)$, then this waiting value is specified by

$$U_f = 0 + \beta E V_f(p),$$

where the expectation $E V_f(p)$ is taken with respect to $G(p)$, capturing uncertainty about future investment prospects.

Substituting (3) and (2) to the firm’s search model yields the Bellman’s functional equation as follows:

$$V_f(p) = \max \left\{ \frac{\pi_e(p) + \beta^2 \rho^c E V_f(p)}{1 - \beta p}, \beta E V_f(p) \right\},$$

which is used by the firm to derive its optimal search strategy as reservation type policy $\hat{p}$: accepting projects with a type $p \leq \hat{p}$ without any delay; rejecting any projects with a type $p > \hat{p}$ to wait in the hope of finding better opportunities. This search model is depicted in figure 1, with parameters $(r, \mu)$ suppressed from (4) to avoid notational clutter. If $R_o > \rho_o$ and the bank’s interest policy is such that $r E_{p \leq \hat{p}}(p) = \rho_o$, then $E_{p \leq \hat{p}} \pi_e(p) - 0 = R_o - \rho_o > 0$ is the per-period average search opportunity cost, and so the firm’s search is costly.

Figure 1 is here.

2.2 Solving the Firm’s Search Model

Since the kernel $\pi_e(p)$ of $V_f(p)$ in (4) is adversely affected by $r$, $U_f$ and $\Pi_f$ are inversely related to $r$. As shown in figure 1, we can identify the range $\Omega_{\mu} = [r, \overline{r}]$ for permissible interest rates $r$ as a search parameter in order for reservation type $\hat{p}$ to be non-degenerate by falling within support $[\underline{p}, 1]$. We assume that riskier firms have a more disperse support such that $\frac{dp(\mu)}{d\mu} < 0$.

The upper bound $\overline{r}$ is an interest rate, above which firm $\mu$ will not invest and which is determined by setting $U_f = \Pi_f(p)$ and $V_f(p) = U_f$ for $p \in [\underline{p}, 1]$. The lower

\(^2\)Eqn (3) can be changed to $U_f = 0 + \beta \left[ \alpha_f E V_f(p) + \alpha^c U_f \right]$ if the firm is assumed to come across a new project only with a probability $\alpha_f$ in the next period. Uncertainty about investment availability of this kind is omitted by setting $\alpha_f = 1$ in the paper for simplicity.
bound \( r \) is an interest rate, at (\& below) which this firm is willing to take any type of project and which is derived from \( U_f = \Pi_f(1) \) and \( V_f(p) = \Pi_f(p) \) for \( p \in [\underline{p}, 1] \). Calculation in appendix (A1) leads to

\[
\tau(\mu) = \frac{R_o}{\underline{p}(\mu)} \propto \frac{1}{\mu} \quad \text{and} \quad \underline{r}(\mu) = \frac{R_o}{1 + \frac{\beta \mu}{1-\beta \mu}} \propto \mu, \tag{5}
\]

where the Taylor expansion of \( \Pi_f(p) \) around \( \mu \) is involved in the calculation of \( \underline{r} \).

Note that \( r > \underline{r} \) makes \( \hat{p} < 1 \). And, \( r \leq \underline{r} \) is a range for \( r \) in which \( \hat{p} = 1 \) so that firm \( \mu \) will take on any type \( p \) of project. Also, note that \( r \leq \tau \) ensures \( \hat{p} \geq \underline{p} \). Especially, firm \( \mu \) will only pick the riskiest type of \( p = \hat{p} = p(\mu) \) from its project population at \( r = \tau(\mu) \). If \( r > \tau \) such that \( \hat{p} \) vanishes, firm \( \mu \) will not borrow for investment. Since \( \tau \) is inversely related to \( \mu \) and \( \underline{r} \) positively to \( \mu \) in (5), riskier firms are willing to pay higher rates and \( \Omega_{\mu_2} \subset \Omega_{\mu_1} \) for \( \mu_1 < \mu_2 \). So, safer firms are more sensitive to interest changes, namely, more averse to high rates and of more prudent at low rates.

As derived in appendix (A2), the optimality condition for the \( \mu \)-type firm’s search model is:

\[
R_o = r \hat{p} + r \beta (1 - \beta \hat{p}) \int_{\underline{p}}^{\hat{p}} \frac{G(p|\mu)}{(1 - \beta p)^2} dp, \tag{6}
\]

which determines this firm’s reservation type for project choice at \( r \): \( \hat{p} = \hat{p}(r, \mu) \). Substituting this back to (6) and differentiating, as shown in appendix (A3), yields the following comparative static results about the effects of \( (r, \mu) \) on reservation policy \( \hat{p} \), as illustrated in figure 2:

\[
\frac{\partial \hat{p}}{\partial r} < 0, \quad \frac{\partial \hat{p}}{\partial \mu} < 0. \tag{7}
\]

Here, we assume \( \frac{\partial G(p|\mu)}{\partial \mu} < 0 \), which implies that \( G \) first-order stochastically dominates (denoted \( f_{\text{fosd}} \)) its lower-mean counterparts.

Figure 2 is here.

**Theorem 1** A rise in interest rate \( r \) increases the riskiness of projects accepted by any firms through a decrease in their reservation type \( \hat{p} \) of investment choice. Riskier firms with higher \( \mu^c \) will take on riskier projects due to lower reservation type \( \hat{p} \) in their investment search.

### 2.3 The Firm’s Repayment Policy

Some firms are clearly more prudent than others, and these differences are reflected in their project choices. Yet firm type and project type may not be of one-to-one
correspondence, and realistically, project types are connected with a firm type only in a probabilistic manner. For example, the likelihood of carrying out a risky project of low \( p \) type by a risky firm of low \( \mu \) type is high, whereas the probability of taking on such a project by a safe firm of high \( \mu \) type is low. To show that different firms have different probabilities of repaying their loan, we regard a firm as characterized by the mean type \( \mu \) of all projects \( p \) that it could possibly undertake.

The variance \( \sigma^2 \) of project distribution \( G(p \mid \mu) \) is assumed as identical across all firms for simplicity. If \( \mu_1 < \mu_2 \) in this setting, firm \( \mu_1 \) is riskier than firm \( \mu_2 \) in the sense of \( G(p \mid \mu_2) \geq G(p \mid \mu_1) \). Heterogeneity of a firm is thus captured uniquely by its average project riskiness \( \mu^c \) \{= \( E(p^c) \)\}, which is a shifting parameter in the search rule \( \tilde{p}(r, \mu) \) as portrayed in figure 2. We assume that the bank knows the distribution \( F(\mu) \) of its potential borrowers by type \( \mu \) in the loan market, but firms may not have or need this information. If banks are different, this distribution is bank-specific with mean \( m = E_F(\mu) \) (i.e., the average risk of a loan portfolio) as one of a bank’s characteristics. \( F(\mu \mid m) \) faced by bank \( m \) is defined on its customer population \( [\mu, \bar{\mu}] \subset [\underline{p}, 1] \), with \( m \) suppressed later.

Based on its project search policy \( \tilde{p} \), a \( \mu \)-type firm’s average success probability \( \tilde{p} \) of accepted projects is defined by

\[
\tilde{p}(r \mid \mu) = E_{p(\mu) \leq p \leq \tilde{p}(r, \mu)}[p \mid G(p \mid \mu)], \quad \text{for } r \in \Omega_{\mu}
\]

where \( \underline{p} \leq \tilde{p} \leq \min(\tilde{p}, \mu) \). From a bank’s perspective, \( \tilde{p}(r \mid \mu) \) is also the firm \( \mu \)'s average rate of loan repayments at interest rate \( r \). It follows that \( \tilde{p} \) is positively related to \( \tilde{p} \) and to \( \mu \), and negatively to \( r \) since

\[
\frac{d\tilde{p}}{d\tilde{p}} = \frac{\hat{g}}{\hat{G}(\tilde{p} - \tilde{p})} > 0, \quad \frac{\partial\tilde{p}}{\partial r} = \frac{d\tilde{p}}{d\tilde{p}} \frac{\partial \tilde{p}}{\partial r} < 0, \quad \frac{\partial\tilde{p}}{\partial \mu} > 0,
\]

where the positive sign of \( \frac{\partial\tilde{p}(r, \mu)}{\partial \mu} \) can be identified by making use of the \( fosd \) concept, as done in appendix (A4). Since \( \frac{\partial \tilde{p}}{\partial \mu} < 0 \) for any given \( \mu \), the moral hazard problem of higher rates takes the form of an adverse incentive effect on firms’ repayment policy.

The firm’s loan repayment policy \( \tilde{p} \) not only carries all the properties of its project search policy \( \tilde{p} \), but also has more informational content than does the latter in the eyes of a bank. Default risk \( p^c \) stochastically ranging in \( [\underline{p}, 1] \) is first confined to \( [\underline{p}, \tilde{p}] \) for any given \( (r, \mu) \) under the firm’s search rule, and then becomes deterministic in the form of \( \tilde{p}^c \), which summarizes the remaining randomness of \( p^c \) \( (\leq \tilde{p}) \) in the firm’s investment action, and thus is of great use for the bank’s decision-making. Therefore, it is better to treat \( \tilde{p}(r \mid \mu) \) than \( \tilde{p}(r, \mu) \) as the firm’s reaction function to the bank’s interest policy, and we will work with \( \tilde{p} \) later on.

### 3 The Bank’s Search for Acceptable Borrowers

This section provides a search technology for a forward-looking bank to determine the acceptability of borrowers on the basis of their type distribution and reaction
functions. Emphasis is placed on the impacts of \textit{ex ante} uncertainty about future loan portfolios on the bank’s customer-selection decision given its interest-setting policy. An optimal selection mechanism is developed in this borrower-type search model to impose credit rationing only on risky firms as desired rather than on safe firms as done in the literature. Adverse selection thus gives way to positive selection under imperfect information, and the moral hazard problem is largely alleviated as well.

\subsection*{3.1 The Bank’s Search Model}

Under limited liability and no collateral, profits \( \pi_b \) a bank expects to receive each period from lending to a \( \mu \)-type firm are given by \( r - \rho_o \) with probability \( \tilde{p}(r \mid \mu) \) and \( 0 - \rho_o \) with probability \( \tilde{p}^c(r \mid \mu) \), where \( \rho_o \) has to be paid by the bank to its depositors, regardless of whether the firm is in default. The bank’s mean profit from this lending, \( \mu \pi_b(r \mid \mu) = r\tilde{p}(r \mid \mu) - \rho_o \), is increasing in \( \mu \) since \( \frac{\partial \tilde{p}}{\partial \mu} > 0 \). At the time of deciding whether to lend firm \( \mu \) a loan, the bank cannot observe the type \( \mu \leq \tilde{p}(r, \mu) \) of a project that it is financing, yet the firm knows \( \mu = p \) at this time (informational asymmetry \textit{ex post}). So, the bank must base its calculation of the distribution \( G(p \mid \mu) \) and average repayment rate \( \tilde{p}(r \mid \mu) \) of this firm.

There are two cross-time uncertainties involved in the bank’s decision-making. The first is that even if the bank may find the type \( \mu \) (\textit{ex post} realization) of a borrower after receiving its loan application, uncertainty about what types \( \mu \) (stochastic) of potential borrowers will arrive still prevails \textit{ex ante}. The bank needs to search for acceptable borrowers based on distribution \( F(\mu) \), which is assumed to be \textit{i.i.d.} over time. The second is that the bank, though perfectly aware at any time of a \( \mu \)-type firm’s project acceptance rule \( \mu \leq \tilde{p}(r, \mu) \), does not know what exact type \( \mu \) (random) of project is to be financed in the future. This uncertainty governed by \( G(p \mid \mu) \) and faced by the bank, along with the investment rule, is already embedded in the firm’s reaction function \( \tilde{p}(r \mid \mu) \) in (8).

A forward-looking bank as a Stackelberg price leader, with a \( \mu \)-type firm’s loan application in hand and its reaction function \( \tilde{p}(r \mid \mu) \) in mind, is confronted with a choice this period of whether to lend this firm a loan while having options to search for other lending opportunities in the subsequent periods. The bank’s search value \( V_b(\mu) \) is defined by a search model \( V_b(\mu) = \max \{ \Pi_b(\mu), U_b \} \), where \( \Pi_b(p) \) is acceptance value and \( U_b \) rejection (or waiting) value. Denote \( \tilde{p}(\mu) \equiv \tilde{p}(r \mid \mu) \) for simplicity.

If the bank chooses to lend firm \( \mu \) a loan this period and behaves optimally in the subsequent periods, then it will receive an acceptance value \( \Pi_b(\mu) \), which satisfies a recursive equation as follows:

\[
\Pi_b(\mu) = \tilde{p}(\mu) \{ r - \rho_o + \beta \Pi_b(\mu) \} + \tilde{p}^c(\mu) \{ 0 - \rho_o + \beta U_b \} . \tag{10}
\]

Since the bank earns a profit equal to \( r - \rho_o > 0 \), as implied by (25)) from loaning to firm \( \mu \) that repays the loan this period, it will provide this firm with refinancing
in the next period, from which onward the process of generating $\Pi_b(\mu)$ repeats itself. If the firm fails in repaying this period, the bank incurs a total loss while having to pay its depositors by $\rho_o$, in which case the bank will return to the market to receive a waiting value $U_b$ from the next period on. Then, rearranging (10) yields

$$\Pi_b(\mu) = \frac{r\tilde{p}(\mu) - \rho_o + \beta \tilde{p}c(\mu) U_b}{1 - \beta \tilde{p}(\mu)}.$$  \hfill (11)

Since $\Pi_b(\mu) > 0$ as shown in appendix (A5), the bank prefers safe borrowers.

The bank acquires a rejection value $U_b$ if it chooses to reject the firm $\mu$’s loan application this period and to look to other applications for better lending opportunities. Then, from equation

$$U_b = - (\rho_o - 1) + \beta EV_b(\mu),$$  \hfill (12)

one sees that the bank has to pay its depositors the interest rate $\rho_o - 1 (> 0)$ for retaining their funds during this waiting period, and is assumed to receive a loan application from a new $\mu$-type firm in the next period to obtain a search value $V_b(\mu)$. Expectation $EV_b(\mu)$ is taken with respect to distribution $F(\mu)$, reflecting uncertain risks and opportunities of future lending. A new firm $\mu$ to show up may be a safe one, which will provide profits $(r - \rho_o > 0)$ for the bank with a high probability $\tilde{p}(\mu)$. Also, this new firm may turn out to be a risky one, and the firm, whatever its type, might default with probability $\tilde{p}c(\mu)$ so as to have the bank incur a total loss $(-\rho_o)$.

Substituting (12) and (11) to the bank’s search model yields the following Bellman’s functional equation:

$$V_b(\mu) = \max \left\{ \frac{r\tilde{p}(\mu) - \rho_o [1 + \beta \tilde{p}c(\mu)] + \beta \tilde{p}c(\mu) [1 + \beta EV_b(\mu)]}{1 - \beta \tilde{p}(\mu)}, \frac{1 - \rho_o + \beta EV_b(\mu)}{1 - \beta \tilde{p}(\mu)} \right\},$$  \hfill (13)

where parameter $r$, suppressed to keep notation neat, remains to be determined in market equilibrium. Since this search model is based on $F(\mu)$, there should be a restriction, namely, $r \leq \tau(\tilde{p})$, to make the interest rate acceptable to all borrowers in $[\mu, \bar{\mu}]$ under consideration. From this model is derived the bank’s optimal search strategy as reservation type policy $\hat{\mu}$: lending a loan to firms with a type $\mu \geq \hat{\mu}$ without any delay; rejecting any firms with a type $\mu < \hat{\mu}$ to process other loan applications \(^3\) or wait for new ones.

\(^3\)A bank can also cope with multiple i.i.d. loan applications each period in this search model, adopt a Poisson arrival process to incorporate random short-time intervals for continuous-time search, and introduce an arrival rate $\alpha_b$ of loan applications to investigate the effects of customer availability. Note that $\alpha_f$ and $\alpha_b$ may be correlated to some degree.
3.2 Solving the Bank’s Search Model

Following the procedure in appendix (A5), we derive the optimality condition below for the bank’s search model:

\[
\tilde{p}(r | \hat{\mu}) - \frac{1}{r} = \beta [1 - \beta \tilde{p}(r | \hat{\mu})] \int_{\hat{\mu}}^{r} \frac{\partial \tilde{p}(r | \mu)}{\partial \mu} \frac{1 - F(m | \mu)}{[1 - \beta \tilde{p}(r | \mu)]^2} d\mu > 0, \tag{14}
\]

which defines the bank’s reservation choice of borrower type: \( \hat{\mu} = \hat{\mu}(r; m) \) (suppress \( m \) below). Note that \( \rho_o \) has been cancelled out of (14). Also, one sees from (14) that \( r\tilde{p}(\hat{\mu}) > 1 \). Then, \( E_{\mu \geq \hat{\mu}} [r\tilde{p}(\mu) - \rho_o] = (1 - \rho_o) = rE_{\mu \geq \hat{\mu}} [\tilde{p}(\mu)] - 1 \geq r\tilde{p}(\hat{\mu}) - 1 > 0 \) is the per-period average search opportunity cost, and so the bank’s search is costly as well.

As shown in appendix (A6), substituting \( \hat{\mu} = \hat{\mu}(r) \) back to (14) and differentiating with respect to \( r \) yields

\[
\frac{d}{dr} \tilde{p}(r | \hat{\mu}(r)) = \frac{\partial \tilde{p}}{\partial \mu} \hat{\mu}'(r) + \frac{\partial \tilde{p}}{\partial r} = -Q(r) < 0, \tag{15}
\]

where \( Q(r) > 0 \) is some quantity in (36), determined by the direct effect of rising interest rates via \( \frac{1}{r} \) (i.e., usual price effect) and their indirect effect via \( \frac{\partial \tilde{p}}{\partial r} \) (i.e., their adverse incentive effect on investment action by borrowers in \([\hat{\mu}, \tilde{\mu}]\)). The search model offers the bank one more policy instrument \( \hat{\mu}(r) \) besides interest rate \( r \), and this new tool for its customer-selection is closely linked to its interest policy with potential moral hazard effects. The customer-selection strategy together with this linkage can be utilized not only to remove adverse selection effects but also to establish a positive selection mechanism for offsetting moral hazard, as explained below.

The total effect \( \frac{d\tilde{p}}{dr} \) in (15) of an interest change on the bank’s reservation demand \( \tilde{p} \) for firms’ repayment rate \( \tilde{p} \) is a negative quantity equal to \(-Q \) and, can be decomposed to the firm’s project-selection effect \( \frac{\partial \tilde{p}}{\partial \mu} \hat{\mu}'(r) \) (negative) and the bank’s customer-selection effect \( \frac{\partial \tilde{p}}{\partial r} \hat{\mu}'(r) \) (depending on the sign of \( \hat{\mu}'(r) \) since \( \frac{\partial \tilde{p}}{\partial r} > 0 \)). Although the sign of \( \hat{\mu}'(r) \) is in general unclear since \( \hat{\mu}'(r) = -\left[ \frac{\partial \tilde{p}}{\partial r} + Q(r) \right] / \frac{\partial \tilde{p}}{\partial \mu} \leq 0 \), one thing is certain in (15) and (36) that \( \frac{\partial \tilde{p}}{\partial r} \hat{\mu}'(r) \to -\frac{k(r)}{r^2} < 0 \) or \( \hat{\mu}'(r) \to 0 \) if the moral hazard problem is a minor one (say, \( \left| \frac{\partial \tilde{p}}{\partial r} \right| \) and \( \left| \frac{\partial \tilde{p}}{\partial \mu} \right| \) → 0). In this case where ”negative” selection is rational with \([\hat{\mu}, \tilde{\mu}]\) widened, the bank will tolerate the including of some riskier firms in its lending portfolio at higher rates. Here, the ”negative” selection is not adverse since more risky firms enter the bank’s lending pool without crowding out safe firms. However, changes in reservation \( \hat{\mu} \) are complicated in other cases when the moral hazard problem is severe. If the bank’s search decision is such that \( \hat{\mu}'(r) > 0 \), it must have refused to lend riskier firms a loan at high rates. In this case where positive selection is optimal with \([\hat{\mu}, \tilde{\mu}]\) narrowed, the bank chooses prudent borrowers...
as a safe outlet for its loans to subdue the adverse incentive effect following a rise in interest rate. We have thus seen that a bank under search can be flexible in its selection of customers given its own choice of interest rates with unavoidable moral hazard effects.

### 3.3 The Bertrand Equilibrium Rate

There are assumed to be several (or many) homogeneous banks in the loan market. These banks are a Bertrand competitor to provide identical, financial (depositing/loaning) services for maximal profits since they freely compete through price (r)-setting in a simultaneous-move game. Thus, this loan market must produce the same equilibrium as a perfectly competitive market in terms of price and profits. All banks share the market and earn zero profit in the long-run equilibrium.

To show the existence of Bertrand equilibrium associated with two-sided search in the market, we denote by \( \Lambda_m \equiv [\underline{r}, \overline{r}] \) the domain of permissible rates to a representative bank (of type \( m \)) such that \( \bar{\mu}(r) \in [\underline{\mu}, \overline{\mu}] \) if \( r \in \Lambda_m \). Realistically, the bank stops lending at (or below) \( \underline{r} \) but will lend everybody at (or above) \( \overline{r} \). We assume that the underlying distributions and parameters are such that the lower bound \( \underline{r} = \underline{r}(\mu) \) can be determined by

\[
\Pi_b(\mu) = U_b(\mu) = \Pi_b(\mu) \quad \text{for } \mu \in [\underline{\mu}, \overline{\mu}]
\]

while the upper bound \( \overline{r}(\mu) \) by setting

\[
\Pi_b(\mu) = V_b(\mu) = \Pi_b(\mu) \quad \text{for } \mu \in [\underline{\mu}, \overline{\mu}].
\]

Denote the bank’s mean profit by lending a \( \mu \)-type firm as

\[
\bar{\pi}(r, \mu) = r \exp(\frac{r}{\rho} - \rho_o) - \rho_o.
\]

It is easy to see that \( \bar{\pi}(r | \overline{r}) \equiv 1 \) and \( \bar{\pi}'(\overline{r}) < 0 \) if \( \frac{\partial \bar{\pi}(r, \mu)}{\partial r} > 0 \) (the bank’s marginal profit is positive at the lowest of permissible interest rates). Suppose \( \overline{r}'(m) > 0 \), then the bank with a lower (\( m^c \)) average risk in its loan pool tends to be more accommodating to low rates.

Intuitively, firms prefer a low rate of interest while having to go risky in their project search if the rate is up. However, a bank is by no means willing to offer an interest rate that is too low to make a non-negative profit, and will no longer be afraid of setting high rates (within firms’ acceptance limits, of course) since its search technology now makes the positive selection possible under imperfect information. In the context of two-sided search, therefore, it is the bank’s reservation supply price and firms’ reservation demand prices which should be binding and prevailing in the market. Take \( \Lambda_{\overline{\pi}} \equiv \Lambda_m \cap \Omega_{\overline{\pi}} = [\underline{r}(\overline{r}), \overline{r}(\overline{r})] \) to be the set of interest rates mutually acceptable by all firms and the bank. Since the bank’s search (13) already includes all firms as potential borrowers by having \( r \leq \overline{r}(\overline{r}) \) in \( \Lambda_{\overline{\pi}} \), adverse selection will not take effect at all in the following Bertrand equilibrium even with the unobservability of firm types \( \mu \) by any bank.

The average repayment rate of all the firms lent a loan under bank search (13)
and the bank’s mean profit from lending to these borrowers are

\[
\tilde{p}(r) = E_{\mu \leq \hat{\mu}} \left[ \tilde{p}(r | \mu) \right] = \frac{\int_{\hat{\mu}}^{\mu} \tilde{p}(r | \mu) \, dF(\mu)}{1 - F(\hat{\mu}(r))}, \\
\hat{R}(r) = r\tilde{p}(r) - \rho_0, \quad \text{for } r \in \Lambda_{\mu}. \quad (16)
\]

Additionally, we assume that \( \frac{\hat{R}}{\rho_0} > \frac{\hat{R}(\mu)}{\mu} (> 1) \). As shown in appendix (A6), the limiting values of bank profit function \( \hat{R}(r) \) are

\[
\hat{R}(r) \bigg|_{r < \mu(\bar{F})} = 0, \quad \hat{R} \big[ \mu(\bar{F}) \big] < 0, \quad \hat{R} \big[ \mu(\bar{F}) \big] > 0. \quad (17)
\]

Clearly, \( \hat{R}(r) \) is continuous everywhere in \( \Lambda_{\mu} \). Thus, curve \( \hat{R}(r) \) must cross the horizontal \( r \)-axis at least once, and there exists at least one \( r \) as an interior solution such that the Bertrand condition \( \hat{R}(r) = 0 \) holds. For Pareto improvement, we take the smallest of those \( r \)’s, if not unique, as the market equilibrium rate of interest, denoted \( r^* = r^*(\rho_0, m) \). As illustrated in figure 3, this solution point cannot appear on the left of \( r(\mu) \) since the bank will not lend and its profit is zero, implying that \( \frac{d\hat{R}}{d\mu} (\mu) < \tilde{p}(r) \), that is, curve \( \frac{d\hat{R}}{d\mu} \) crosses curve \( \tilde{p}(r) \) from above at point \( r^* \).

Figure 3 is here.

**Theorem 2**  
The search model under imperfect information equips a bank with a flexible rule for its optimal customer-selection to subdue moral hazard problems associated with its own interest-rate choice. There exists a Bertrand interest rate in search equilibrium that wipes out adverse selection even under the ex ante unknown firm types.

### 4 Equilibrium Market Segmentation

This section analyzes how the repeated use of its search technology enables the bank to rationally group its customers based on its imperfect knowledge of their risk characteristics and to eliminate the need for rationing credit. The two-sided search in this framework is shown to best feature the interactions between different firms and homogeneous banks in the credit market. The interaction yields firm groups’ repayment policies, a typical bank’s group-discriminating interest rates, and an optimal market segmentation in the Bertrand equilibrium. An equilibrium lending rate arises within each firm group from competition among identical banks, which are all of type \( m \) and \( m' \).

\[^4\]One may also consider a case of different banks (e.g., indexed by \( j \)) characterized by different distributions \( F_j(\mu) \) or simply \( F(\mu | m') \), by different customer arrival rates \( \alpha_j \), and/or by different depositing rates \( \rho_0' \). Then, the Bertrand competition should take place among identical banks within their group.
compete to provide loan financing for firms in this group. This ex ante segmentation makes it possible that different borrowers will pay different rates for their different default risks.

4.1 Grouping Loan Customers

If a bank employs model (13) to search for borrower types, then safer firms with a type \( \mu \in [\tilde{\mu} (r), \overline{\mu}] \) are kept in its loan pool at interest rate \( r \), and positive selection instead of adverse selection takes effect. However, if the bank were to use this model only once, then riskier firms with a type \( \mu \in [\underline{\mu}, \tilde{\mu} (r)] \) would be excluded from borrowing at that rate \( r \), a typical case of credit rationing; nevertheless, this exclusion is different from the literature \(^5\) in which safer borrowers are subjected to credit rationing. Then, the bank can use the search model for enough times in order to include those riskier firms in its loan portfolio on the stricter terms of lending, given that they are willing to pay higher rates for borrowing.

The advantage of repeatedly using the search technology is enabling the bank to optimally group its customers based on its imperfect knowledge of their hidden risk characteristics. This can be achieved if the distribution underlying expectation \( EV_b (\mu) \) in (13) is to be changed accordingly. For \( i = \{1, 2, \cdots, n\} \) and \( \overline{\mu} \equiv \overline{\mu} \), denote \( \Lambda_{\tilde{\mu}^*_{i-1} \tilde{\mu}^*_i} = [\mu (\tilde{\mu}^*_{i-1}), \tilde{\mu} (\tilde{\mu}^*_{i-1})] \) as the set of interest rates that make the bank be willing to lend and all firms in \( [\mu, \tilde{\mu}^*_i] \) be willing to borrow in the bank’s \( i \)th search process. The distribution of firms attracted by the bank’s first search selection, denoted \( F_1 (\mu) \), is the original distribution, i.e., \( F_1 (\mu) = F (\mu) \) for \( \mu \in [\underline{\mu}, \tilde{\mu}_0] \), the whole support. As mentioned before, \( r \in \Lambda_{\tilde{\mu}^*_{i-1} \tilde{\mu}^*_i} \) is the restriction on interest rate for this model. The bank’s reservation demand for the borrower types of this group, \( \tilde{\mu} \) in (13), is now denoted by \( \tilde{\mu}_1 (r) \) (< \( \tilde{\mu}^*_1 \)). This first search model is to identify the first (or the safest) group of firms with types \( \mu \in [\tilde{\mu}^*_1, \tilde{\mu}^*_0] \), where \( \tilde{\mu}^*_1 = \tilde{\mu}_1 (r^*_1) \), and the Bertrand interest rate \( r^*_1 \) for this group has been determined as \( r^*_1 (\rho_0, m) \) earlier.

All other search models are identical to the first one except for underlying distributions used. In general, the bank’s \( i \)th search model is based on a truncated distribution \( F_i (\mu) \) of firm types \( \mu \) in \( [\mu, \tilde{\mu}^*_i] \), a shrinking support due to the preceding search selection, where

\[
F_i (\mu) = F (\mu) / F (\tilde{\mu}^*_{i-1}), \quad \text{for} \quad i = 2, \cdots, n;
\]

\( \tilde{\mu}^*_{i-1} \) is such that \( \tilde{\mu}^*_{i-1} \) is close to \( \mu \) by some reasonable stopping rule, and we simply set \( \tilde{\mu}^*_{i-1} = \mu \) for convenience. The interest restriction now becomes \( r \in \Lambda_{\tilde{\mu}^*_{i-1} \tilde{\mu}^*_{i}} \), keeping all remaining firms in \( [\mu, \tilde{\mu}^*_{i-1}] \) to willingly undergo the bank’s \( i \)th-round search selection. The bank’s reservation demand for the borrower types of the \( i \)th

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\(^5\)See Stiglitz and Weiss (1981) for statements in theorems 2, 4 and the integral interval in function (7), where \( \theta \) is meant to be the borrower type while also denoting the project risk elsewhere. Also, see Chan and Thakor (1987) for concluding remarks.
group is denoted by \( \hat{\mu}_i (r) \) \( (< \hat{\mu}_{i-1}^* \) \). This \( i \)th search model is used to identify the \( i \)th group of firms with types \( \mu \in [\hat{\mu}_i, \hat{\mu}_{i-1}^*] \) \( \equiv \Theta_i \) for \( i = \{2, \ldots, n\} \), and \( \Theta_1 = [\hat{\mu}_1^*, \hat{\mu}_0] \), where \( \hat{\mu}_i^* \equiv \hat{\mu}_i (r_i^*) \), and the Bertrand equilibrium \( r_i^* \) has yet to be determined for this group.

Since \( F_i (\mu) > F_{i-1} (\mu) \) in \( (\mu, \hat{\mu}_{i-2}^*) \) and \( F_i (\mu) = F_{i-1} (\mu) \) in \( \{\mu\} \cup [\hat{\mu}_{i-2}^*, \hat{\mu}_{i-1}^*] \) for \( i = \{2, \ldots, n\} \) as shown in Figure 4, it follows that

\[
F_1 (\mu) \overset{fosd}{\succ} F_2 (\mu) \overset{fosd}{\succ} \cdots \overset{fosd}{\succ} F_n (\mu),
\]

Denote \( m_i = E_{F_i} (\mu) \) as the mean of distribution \( F_i (\mu) \), and \( m_1 = m \). It follows from the necessary condition of \( fosd \) that \( m_1 > m_2 > \cdots > m_n \).

Figure 4 is here.

One can prove by contradiction that \( n \) is a finite number. Suppose that \( n \to \infty \), then the sum of lengths of type intervals of \( n \) groups would be \( \sum_{i=1}^{n} (\hat{\mu}_{i-1}^* - \hat{\mu}_i^*) \to \infty \), a contradiction to \( \mu - \bar{\mu} < 1 - p < 1 \).

**Theorem 3** A search technology provides the forward-looking bank with a policy instrument for its positive selection of loan customers. Through repeated search by the bank, different firms can be partitioned into a finite number of lending groups given their hidden risk characteristics.

### 4.2 The Firm Group’s Repayment Policy

To see how various firms respond to the bank’s search/grouping behavior, we define a new set of distributions for any given \( r \in \Lambda_{\hat{\mu}_{i-1}^* \hat{\mu}_{i-1}^*} \),

\[
H_i (\mu | r) = \frac{F (\mu) - F (\hat{\mu}_i (r))}{F (\hat{\mu}_{i-1}^*) - F (\hat{\mu}_i (r))}, \quad \begin{cases} \text{for } \mu \in [\hat{\mu}_1 (r), \hat{\mu}_0] & \text{as } i = 1 \\
\text{for } \mu \in [\hat{\mu}_i (r), \hat{\mu}_{i-1}^*] & \text{and } i = 2, \ldots, n \end{cases}
\]

Note that \( H_i (\mu) \) (suppress \( r \)) has to do with the mean \( m \) of original distribution \( F (\mu) \), and the restriction \( r \leq \hat{\mu}_{i-1}^* \) contained in \( \Lambda_{\hat{\mu}_{i-1}^* \hat{\mu}_{i-1}^*} \) is crucial to avoiding potential adverse selection in the Bertrand equilibrium given the unobservability of \( \mu \) in loan pool \([\mu, \hat{\mu}_{i-1}^*]\) with a possibly high average risk \( m_i^c \).

In general, associated with the bank’s \( i \)-th round search is the \( i \)th firm group’s average repayment rate \( \tilde{p}_i (r) \), which is the reaction function of this group to the bank’s grouping activity at interest rate \( r \). Then, by definition in (18) and (20), \( \tilde{p}_i (r) \) and the bank’s mean profit function \( \tilde{R}_i (r) \) from lending to group \( i \) should be

\[
\tilde{p}_i (r) = E_{\hat{\mu}_i (r) \leq \mu < \hat{\mu}_{i-1}^*} [\tilde{p} (r | \mu) | F_i] = \frac{\int_{\hat{\mu}_i (r)}^{\hat{\mu}_{i-1}^*} \tilde{p} (r | \mu) dF_i (\mu)}{1 - F_i (\hat{\mu}_i (r))} = \frac{\int_{\hat{\mu}_i (r)}^{\hat{\mu}_{i-1}^*} \tilde{p} (r | \mu) d\mu}{1 - F_i (\hat{\mu}_i (r))} = E_{H_i (\mu) \tilde{p} (r | \mu)}
\]

(21)
\[ \tilde{R}_i (r) = r \tilde{p}_i (r) - \rho_o , \quad \text{for} \quad r \in \Lambda \mu_{i-1}^{*} \mu_{i-1}^{*} \quad \text{and} \quad \forall \ i, \]

where \( F_i \) used here for \( \tilde{p}_i \) is the same as that for \( EV_b \) in the \( i \)th search model. In particular (as \( i = 1 \)), \( \tilde{p}_1 (r) \) and \( \tilde{R}_1 (r) \) in (21) are exactly identical to \( \tilde{p} (r) \) and \( \tilde{R} (r) \) in (16), respectively.

To clearly reflect the strategic interactions between this group’s repayment policy and the bank’s search/grouping activity for the \( i \)th time, we rewrite \( \tilde{p}_i \) in its full form as
\[
\tilde{p}_i (r) = E_{\mu_i \leq \mu < \mu_{i-1}^{*}} \left\{ E_{\mu \leq \mu_i \leq \tilde{p}_i (r, \mu)} \left[ G \ (p \ | \ \mu) \right] \ | \ F_i (\mu) \right\}. \tag{22}
\]

Indeed, this expression pinpoints the major features of a two-sided search interaction in terms of \( \tilde{p} \) and \( \tilde{\mu}_i \), with interest rate \( r \) as a transmission mechanism to link together all decisions of both parties and, distributions \( G \) and \( F \) as information structure for the two uncertainties. Also, firm group \( i \) in making its repayment decision takes into consideration not only the bank’s search reservation policies for its own group, \( \tilde{\mu}_i (r) \) at interest rate \( r \), but also for another group, \( \tilde{\mu}_{i-1} (r) \) at equilibrium rate \( r_{i-1}^{*} \).

As shown in appendix (A7), the sign of \( \tilde{p}_i (r) \) is ambiguous since \( \tilde{\mu}_i (r) \) has no definite sign due to the unknown magnitude of moral hazard relative to other factors in this general setting. If the moral hazard of an individual firm’s action is so small that \( \tilde{\mu}_i (r) < 0 \), then \( \tilde{p}_i (r) < 0 \) as usual in (38), a case where the adverse incentive effect on the group’s action is caused by the bank’s ”negative” selection. Still, \( \tilde{p}_i (r) \geq 0 \) for \( \tilde{\mu}_i (r) > 0 \), yet this suggests a possibility that a high rate may boost group repayments (i.e., \( \tilde{p}_i (r) > 0 \)) if the bank raises its reservation demand for firm type at high rates (i.e., \( \tilde{\mu}_i (r) > 0 \)), a case where the positive incentive effect on the group’s action is attributable to the bank’s positive selection. If this is true, then a high interest rate, though definitely having an adverse incentive effect on any individual firm’s action, will exert a favorable impact on group repayments since the group now consists of fewer firms with higher types \( \{ r \uparrow \Rightarrow \ 	ilde{\mu}_i (r) \uparrow \leq \mu \leq \tilde{\mu} \Rightarrow \tilde{p}_i (r) \uparrow \} \). This is obviously an advantage created by grouping customers in the bank’s interests.

Figure 5 is here.

By extending the support of each distribution \( H_i (\mu) \) to domain \( [\mu, \tilde{\mu}] \), we find, as shown in figure 5, that for \( i = \{ 2, 3, \ldots, n \} \),
\[
H_i (\mu) > H_{i-1} (\mu) \quad \text{in} \quad \mu \in (\tilde{\mu}_{i-2}^{*}, \tilde{\mu}_{i-2}^{*}) \quad \text{and} \quad H_i (\mu) = H_{i-1} (\mu) \quad \text{in} \quad \left[ \mu, \tilde{\mu} \right] \setminus (\tilde{\mu}_{i-1}^{*}, \tilde{\mu}_{i-1}^{*}).
\]

Therefore,
\[
H_1 (\mu) \succ \ H_2 (\mu) \succ \cdots \succ H_n (\mu). \tag{23}
\]

Since \( \frac{\partial p}{\partial \mu} > 0 \), it follows from (21) and (23) that riskier groups have a lower average rate of loan repayments, and vice versa:
\[
\tilde{p}_1 (r) > \tilde{p}_2 (r) > \cdots > \tilde{p}_n (r). \tag{24}
\]
4.3 Group-Discriminating Interest Policy

We now show how the bank incorporates a firm group’s repayment policy into its choice of interest rates. Note that the bank may make a profit or otherwise from loaning to a particular firm in a group (i.e. \( \bar{R}(r, \mu) \geq 0 \) for a \( \mu \in \Theta_i \)), but must earn zero mean profit (i.e. \( \bar{R}_i(r) = 0 \)) from lending to this group as a whole in the Bertrand equilibrium. Under this condition, the bank can charge each group a different rate of interest, which is derived from

\[
R_i(r) = \rho_o \quad \Rightarrow \quad r_i^* = r^*(\rho_o, m_i). \tag{25}
\]

Note that \( \frac{R_i}{\rho_o} > \frac{p(\mu)}{p(\mu)} \) implies \( \frac{R_i}{\rho_o} > \frac{p(\mu_{i-1})}{p(\mu)} \) for \( \forall i \) since \( p(\mu) > 0 \). It then follows that \( r_i^* \in \Lambda \mu_{i-1}, \mu_{i-1} \) does exist as an interior solution to the equation in (25) for a reason similar to (17). Obviously, the group interest rate is higher than the deposit rate since \( r_i^* = \rho_o/\hat{p}_i > \rho_o \).

Once \( r_i^* \) is worked out, the bank can calculate \( \hat{\mu}_i = \hat{\mu}_i(r_i^*) \) to move to the next search selection, then derives the average repayment rate \( \hat{p}_{i+1}(r) \) of group \( i+1 \), and uses the Bertrand condition again to determine the next equilibrium rate \( r_{i+1}^* \). This process continues successively until the last group rate \( r_n^* \).

With these equilibrium rates, the bank can eventually partition all firms of different risks \( \mu \) into lending groups in such a way that \( \hat{\mu}_0 = \mu, \hat{\mu}_n = \mu \), and

\[
\mu < \hat{\mu}_{n-1} < \cdots < \hat{\mu}_1 < \mu. \tag{26}
\]

See figure 6 for the interest rate domains permissible to various groups associated with this equilibrium partition of different borrowers: \([\mu, \mu] = \cup_{i=1}^n \Theta_i \).

Figure 6 is here.

Because curve \( \frac{\hat{p}_i}{r} \) is decreasing in \( r \) and \( \frac{\hat{p}_i}{r} \leq \frac{\hat{p}_i}{r} \) (as mentioned before), we obtain from (24) and (25) the bank’s group-discriminating interest policy in the Bertrand equilibria:

\[
r_1^* < r_2^* < \cdots < r_n^*, \tag{27}
\]

regardless of the slope of \( \bar{p}_i(r) \), which can be shown in a way similar to figure 3. Therefore, the bank must charge a riskier (or safer) group a higher (or lower) rate of interest.

Denote \( \tilde{p}_u(r) = E_{\mu \in \mu, r} [\bar{p}(r | \mu) | F] \) as the overall repayment rate across all firms, and \( r_u^* = r_u^*(\rho_o, m) \) as the non-discriminating Bertrand rate such that \( r_u^* \tilde{p}_u(r_u^*) = \rho_o \) without bank search. In this case, very safe firms must be made worse off because of being forced to substantially subsidize very risky firms under the pooling of very different default risks, and adverse selection can possibly arise and lead to market failure if the average borrowers of the loan pool are too risky. It can be shown by comparing \( \bar{p}_u(r) \) and \( \tilde{p}_i(r) \) that \( r_1^* < \cdots < r_u^* < \cdots < r_n^* \). This relationship indicates that there will be no cross-group subsidies because of different group rates,
though cross-subsidization still exists among members within a group. Thus, group-discrimination is superior to non-discrimination in terms of reduction in moral hazard and the elimination of adverse selection.

**Theorem 4** The riskier the types of group members, the lower the group’s average rate of loan repayments at any interest rate and the higher will be the corresponding group rate in the Bertrand equilibrium. Additionally, the moral hazard problem is effectively mitigated since there are no longer heavy cross-subsidies among very different firms.

### 4.4 Why This Model Works in Practice

It is important to distinguish an *ex post* choice-making from the *ex ante* decision rule derived from a search model. The optimal reservation policy such as $\tilde{p}$ and $\tilde{\mu}$ is a decision rule, which must be determined based on uncertain information such as $G$ and $F$ before any realization of $p$ and $\mu$. A particular realization such as $p$ and $\mu$ matters only in a choice-making later on, but is of no use for establishing in advance the decision rule. Also, interest rates $r^*_i$ in (25) and equilibrium partitioning $\mu^*_i$ in (26) are determined *ex ante*, and firms are all aware of these credit policies so that the repayment decision rule of firm $\mu$, $\tilde{p}^*_\mu (\mu) = \tilde{p} (r^*_i | \mu \in \Theta_i)$, ensues immediately.

In practice, at the initial stage before going to the market, no choices can be made by any agents while certain decision rules need to be established based on prior information. After entering the market, a $\mu$-type firm comes across a $p$-type project and chooses to accept it only if its type $p$ is no greater than $\tilde{p}^*_i (\mu)$, in which case the firm then goes to a bank to apply for loans. The bank does not have to employ very costly screening devices to reveal the firm’s exact type $\mu$, and all it needs to do is to locate which category $\Theta_i$ this firm falls into while the firm, if it is a safe one, may also signal its type in some economical way. Finally, both parties sign a financing contract at the right interest rate $r^*_i$. As a matter of fact, a long-term stable relationship in credit business between the bank and its customers makes it easy to find out their rough types by information accumulation and interaction experiences.

This viable two-sided search model has some obvious advantages over other alternative. An informal welfare analysis is this. As indicated above, group-discrimination in this model is preferred to non-discrimination due to the latter’s potential adverse selection and severe moral hazard under heavy cross-subsidization. Also, group-discrimination may not be worse than the scheme of perfectly-discriminating rates since the latter involving no bank search rests on revealing a firm’s exact type at high costs. Usually, there are many different firms in the loan pool, it may be very costly to charge so many different rates that are very demanding in screening, writing, monitoring, and enforcing contracts. So, the perfect-discrimination scheme is largely infeasible. Moreover, the non- and perfect-discrimination interest settings do not consider uncertainty about firm types and its impact on a bank decisions about interest rate and customer selection. Finally, safe firms under group-discrimination,
though still having to subsidize risky peers within their group to a lesser extent, can enjoy some benefits from pooling risks. The trade-off of risk-pooling against cross-subsidy by firms (if very risk averse) may make group-discrimination outweigh perfect-discrimination, and under the former scheme a bank with forward-looking perspective concerning lending uncertainty charges a large number of firms only a small number of rates in an economical manner.

5 Conclusion

This paper assumes that a firm has to borrow for production, that the riskiness of a project to be undertaken is observable by the firm but not by a bank as its financier, and that riskier projects tend to be more profitable under limited liability and no collateral. Given the investment choice of firms equipped with a search technology, the moral hazard problem is shown to be significant under informational deficiencies about project risks, in the sense that any firm’s average rate of loan repayments decreases with higher interest rates. If the bank earns zero profit in search equilibrium, there will not be adverse selection effects thanks to the bank’s flexible utilization of *positive* or "negative" selection even with the *ex ante* unobservability of firms’ types. This paper proceeds by assuming that a bank faces *ex ante* uncertainty about lending risks and has a limited ability to ascertain *ex post* the types of potential borrowers. Taking into account in advance its customers’ repayment policy, the bank can also make use of a search technology to integrate its choice of interest rates with its selection of customers. The repeated use of its search model enables the bank to rationally group borrowers according to its incomplete knowledge of their risk characteristics, to charge different groups a different interest rate, and to eliminate credit rationing eventually. Risky borrowers are thus forced to pay high rates they are supposed to, so that heavy cross-subsidies from very safe firms to very risky ones are lessened, and the moral hazard problem is then largely alleviated. Therefore, high rates can still be used to subdue default risk while market failure is avoided.

6 Appendix

(A1) The property of $\Pi_f(p)$ being decreasing holds because

$$
\Pi'_f(p) = \frac{\beta (R_o - \beta p U_f) - r}{(1 - \beta p)^2} - \frac{r (1 - \beta \bar{p})}{(1 - \beta p)^2} < 0.
$$

(28)

To determine $\bar{r}$, combining $U_f = \Pi_f(1)$ with (2) leads to $U_f = \frac{R_o - r}{\beta p}$. Substituting $V_f(p) = \Pi_f(p)$ into (3) yields $U_f = \beta E (R_o - \frac{p - \mu}{\sigma^2} + \beta p U_f) (1 - \beta p)^{-1}$. Noting $\bar{p} = 1$, recalling from (28) that $\Pi'_f(\mu) = -2 \beta \sigma^2 \sigma (1 - \beta \mu)^{-3} \text{ and } E (p - \mu)^2 = \sigma^2$, second-order Taylor expanding $\Pi_f(p)$ in (2) around $\mu$, and taking expectation with respect
to \( p \), we change \( U_f = \beta E \Pi_f (p) \) to

\[
U_f \approx \beta E_p \left\{ \Pi_f (\mu) + \Pi'_f (\mu) (p - \mu) + \frac{1}{2} \Pi''_f (\mu) (p - \mu)^2 \right\}
\]

\[
= \frac{\beta}{1 - \beta \mu} \left( R_o - r \mu + \beta \mu^2 U_f \right) - \frac{\beta^2 \beta \mu \sigma^2 r}{(1 - \beta \mu)^3},
\]

\[
U_f = \frac{\beta (1 - \beta \mu)^2 R_o - r \beta \left[ \mu (1 - \beta \mu)^2 + \beta^2 \sigma^2 \right]}{\beta^2 (1 - \beta \mu)^2 (1 + \beta \mu^2)}.
\]

Substituting \( U_f = \frac{R_o - \beta \mu}{\beta \mu} \) to the above yields

\[
\bar{r} (\mu, \sigma^2) = \frac{R_o}{1 + \frac{\beta \mu^2}{1 - \beta \mu} - \frac{\beta^2 \beta \mu \sigma^2}{(1 - \beta \mu)^3}},
\]

which reduces to (5) in the paper if setting \( \sigma^2 = 0 \) in (29). This is equivalent to using the first-order Taylor expansion to obtain \( \bar{r} (\mu) \). Note that \( \sigma^2 \) is small if \( p \) is concentrated around \( \mu \), and the Taylor approximation becomes better the smaller \( \sigma^2 \) gets.

To determine \( \bar{r} \), substituting \( V_f (p) = U_f \) to (3) leads to \( U_f = 0 \). Evaluating (2) at \( \mu \), substituting it to \( U_f = \Pi_f (\mu) \) and using \( U_f = 0 \), yields \( R_o - \beta \mu = 0 \). We obtain \( \bar{r} (\mu) = \frac{R_o}{\beta \mu} \). Clearly, \( \bar{r} \propto 1/\mu \) since \( \mu' (p) > 0 \). Denote \( \bar{r} (\mu) \equiv \frac{R_o}{1 + \xi} \). Since \( \xi (\mu) = -\frac{\beta \mu^2}{(1 - \beta \mu)^3} < 0 \), one sees \( \bar{r} \propto \mu \).

(A2) From \( \Pi_f (\hat{p}) = U_f \) for \( \hat{p} \in \Omega_\mu \), it follows that

\[
\pi_e (\hat{p}) = \beta \bar{c} U_f \implies R_o - \beta \bar{c} U_f = \hat{r} \hat{p},
\]

and \( \pi_e (p) \geq \beta \bar{c} U_f \) (\( > 0 \) to be shown in (31)) for \( p \leq \hat{p} \). Calculating \( U_f \) in (3) yields

\[
\beta^{-1} U_f = \beta E \left[ \Pi_f (p) dG (p) + \frac{1}{\hat{p}} U_f dG (p) \right]
\]

\[
= \frac{1}{\hat{p}} \int_{\hat{p}}^{\hat{p}} \left[ \Pi_f (p) - U_f \right] dG (p) + U_f \]

\[
= - \int_{\hat{p}}^{\hat{p}} G (p) \Pi'_f (p) dp + U_f \]

\[
= \hat{r} (1 - \beta \hat{p}) \int_{\hat{p}}^{\hat{p}} \frac{G (p)}{(1 - \beta p)^2} dp + U_f,
\]

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\[ \beta'U_f = \beta r (1 - \beta \hat{p}) \int_{\mathbb{P}} \frac{G(p)}{(1 - \beta p)^2} dp > 0. \] (31)

Combining (30) with (31) offers (6) in the text.

(A3) Applying the Leibniz rule and the implicit function theorem to (6) yields

\[ Y(\hat{p}) \frac{\partial \hat{p}}{\partial r} = - \left\{ \hat{p} + \beta (1 - \beta \hat{p}) \int_{\mathbb{P}} \frac{G(p)}{(1 - \beta p)^2} dp \right\} < 0, \] (32)
\[ Y(\hat{p}) \frac{\partial \hat{p}}{\partial \mu} = - \left\{ r \beta (1 - \beta \hat{p}) \int_{\mathbb{P}} \frac{\partial G(p | \mu)}{\partial \mu} \frac{dp}{(1 - \beta p)^2} \right\} > 0, \]

where \( Y(\hat{p}) = r + r^2 \int_{\mathbb{P}} \frac{dG(p)}{1 - \beta p} > 0 \) and

\[ \frac{\partial G(p | \mu)}{\partial \mu} < 0, \quad \mu_1 < \mu_2, \quad G(p | \mu_1) > G(p | \mu_2) \Rightarrow G(p | \mu_2) \overset{fosd}{\gtrless} G(p | \mu_1). \]

(A4) To prove \( \frac{\partial \hat{p}(r, \mu)}{\partial \mu} > 0 \) in (9) by using \( fosd \), suppose that there are two firms characterized by \( G_i(p) = G(p | \mu_i) \) for \( i = \{1, 2\} \), with supports \([p_1, 1]\) and \([p_2, 1]\). Assume that \( \mu_1 < \mu_2, p_1 < p_2 \), and each distribution is bell-shaped and symmetric around its mean. Suppress \( r \), denote \( \hat{p}_j = \hat{p}(\bullet, \mu_j) \) and

\[ \frac{g_i(p | \mu_i)}{G_i(\hat{p}_j | \mu_i)} = \frac{g_i(p)}{G_i(\hat{p}_j)} = g_i(p | \hat{p}_j), \quad \text{with support } \left[ p_1, \hat{p}_j \right], \]
\[ \int_{\mathbb{P}} g_i(p | \hat{p}_j) dp = G_i(p | \hat{p}_j), \quad \text{for } i, j = 1, 2 \text{ and } p \in \left[ p_j, \hat{p}_j \right]. \]

When \( i = j \), denote \( \int_{\mathbb{P}} \hat{p}_i pdG_i(p | \hat{p}_i) = E_i(p | \hat{p}_i) = \hat{p}(\mu_i) = \hat{p}_i. \)

Figure 7 is here.

Since \( \frac{\partial \hat{p}(r, \mu)}{\partial \mu} > 0 \), one sees \( \hat{p}_1 < \hat{p}_2 \). Then, we know from figure 7 that

\[ g_1(p | \hat{p}_1) = \frac{g_1(p)}{G_1(\hat{p}_1)} > \frac{g_1(p)}{G_1(\hat{p}_2)} = g_1(p | \hat{p}_2), \quad \text{for } p \in \left[ p_1, \hat{p}_1 \right]; \]
\[ g_1(p | \hat{p}_1) = 0 < g_1(p | \hat{p}_2), \quad \text{for } p \in (\hat{p}_1, \hat{p}_2). \]

This implies \( G_1(p | \hat{p}_1) \overset{fosd}{>} G_1(p | \hat{p}_2) \) in \( \left( p_1, \hat{p}_2 \right) \). Now compare \( g_1(p | \hat{p}_2) \) with \( g_2(p | \hat{p}_2) \). The supports of both densities have the same upper bound \( \hat{p}_2 \), but the
lower bounds are different in the form of \( p_1 < p_2 \). Therefore, there must exist some cut-off value \( p_c \), as shown in Figure 7, such that

\[
g_1 (p | \hat{p}_2) > g_2 (p | \hat{p}_2), \quad \text{for} \ p \in [p_1, p_c),
\]

\[
g_1 (p | \hat{p}_2) < g_2 (p | \hat{p}_2), \quad \text{for} \ p \in (p_c, \hat{p}_2],
\]

which implies \( G_1 (p | \hat{p}_2) > G_2 (p | \hat{p}_2) \) in \( (p_1, \hat{p}_2) \). By transitivity, if follows that

\[
G_1 (p | \hat{p}_1) > G_2 (p | \hat{p}_2) \quad \Rightarrow \quad G_2 (p | \hat{p}_2) \Rightarrow G_1 (p | \hat{p}_1).
\]

The necessary condition of \( \text{fossd} \) asserts that \( E_2 (p | \hat{p}_2) > E_1 (p | \hat{p}_1) \) or \( \tilde{p} (\mu_2) > \tilde{p} (\mu_1) \), and hence \( \frac{\partial \tilde{p} (r | \mu)}{\partial \mu} = \frac{\partial \tilde{p} (\mu | \mu)}{\partial \mu} > 0 \).

(A5) Denote \( \tilde{p} \equiv \tilde{p} (r | \tilde{\mu} (r)) \). The property of \( \Pi_b (\mu) \) being increasing holds since

\[
\Pi'_b (\mu) = \frac{\partial \tilde{p} r - \beta (\rho_o + \beta U_b)}{\partial \mu} \bigg|_{r = 1} \partial \tilde{p} \bigg( 1 - \beta \tilde{p} \bigg) r \bigg|_{\beta \tilde{p}} > 0, \tag{33}
\]

From \( \Pi_b (\hat{\mu}) = U_b \), it follows that

\[
r \tilde{p} (\hat{\mu}) = \beta U_b + \rho_o. \tag{34}
\]

By the same token as used for (31), applying \( \Pi_b (\hat{\mu}) = U_b, F (\mu) = 1 \) and integration by parts, one can calculate \( U_b \) in (12) and obtains

\[
\beta U_b + \rho_o - 1 = \beta r \bigg( 1 - \beta \tilde{p} \bigg) \int_{\tilde{p}}^{\pi} \frac{\partial \tilde{p} \frac{1 - F}{\partial \mu (1 - \beta \tilde{p})^2}}{d \mu} > 0. \tag{35}
\]

Combining (34) with (35) to eliminate \( U_b \) leads to (14) in the text.

(A6) The expression of \( \frac{\partial \tilde{p}}{\partial \mu} \) in (15) contains

\[
Q (r) \equiv k (r) \left\{ \frac{1}{r^2} - \beta (1 - \beta \tilde{p}) \int_{\tilde{p}}^{\pi} \frac{\partial \tilde{p} \frac{d F}{\partial r (1 - \beta \tilde{p})^2}}{d \mu} \right\} > 0, \tag{36}
\]

where \( \tilde{F} \equiv F (\hat{\mu}) \), \( 1 > \frac{\beta}{r} > 0 \) and \( k (r) \equiv (1 - \beta \tilde{p}) \left[ 1 - \frac{\beta}{r} + \beta (1 - \tilde{F}) \right]^{-1} > 0 \).

The limiting values of \( \tilde{p} (r) \) at and beyond the bounds of \( \Lambda_{\mu r} \) are

\[
\lim_{r \to r' (r' \in (\mu/\mu))} \tilde{p} (r) = \lim_{\tilde{\mu} \to \mu} \frac{\int_{\tilde{\mu} (r)}^{\pi} \tilde{p} (r' | \mu) d F (\mu)}{1 - F (\hat{\mu} (r))} \quad \text{L'hopital} \quad \tilde{p} (r' | \pi), \tag{37}
\]

\[
\lim_{r \to r'' (r'' \in (\mu/\mu))} \tilde{p} (r) = \lim_{\tilde{\mu} \to \mu''} \frac{\int_{\tilde{\mu} (r)}^{\pi} \tilde{p} (r | \mu) d F (\mu)}{1 - F (\hat{\mu} (r))} \quad |_{\tilde{\mu} < \mu'' < \pi} > \tilde{p} (\pi (\mu) | \mu'') > \tilde{p} (\mu'').
\]

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From (??) and (5), and under \( \frac{R_o}{\rho_o} > \frac{\rho(\mathbf{p})}{p(\mu)} \), one sees that the limiting levels of \( \bar{R}(r) \) are

\[
\lim_{r \to r^0} \bar{R}(r) = 0 \quad \text{because the bank will not lend},
\]

\[
\lim_{r \to r^0} \bar{R}(r) = \bar{p}(\mathbf{p}) \frac{\rho(\mu)}{p(\mu)} - \rho_o = 1 - \rho_o < 0,
\]

\[
\lim_{r \to r^0} \bar{R}(r) > \bar{p}(\mathbf{p}) \frac{\rho''(\mu)}{p(\mu)} - \rho_o = \frac{R_o}{\rho(\mathbf{p})} p(\mu) - \rho_o > 0.
\]

(A7) Since \( \frac{\partial \bar{p}}{\partial \mu} > 0 \), it follows that \( \bar{p}(r | \tilde{\mu}_i(r)) \leq E_{\tilde{\mu}_i(r) \in \mu < \tilde{\mu}_i} \{\bar{p}(r | \mu) | F_i\} < \bar{p}(r | \tilde{\mu}_{i-1}) \), or simply \( \tilde{\bar{p}}_i > \tilde{\bar{p}}_i \{\equiv \bar{p}(r | \tilde{\mu}_i(r))\} \). Since \( \frac{\partial \bar{p}}{\partial r} < 0 \), one sees

\[
\tilde{\bar{p}}_i'(r) = (\tilde{\bar{p}}_i - \tilde{\bar{p}}_i) \frac{f_i(\tilde{\mu}_i)}{1 - F_i(\tilde{\mu}_i)} \tilde{\bar{p}}_i'(r) + \int^{\tilde{\mu}_{i-1}} \frac{\partial \bar{p}}{\partial r} dF_i(\mu) \leq 0,
\]

which must be negative if \( \tilde{\bar{p}}_i'(r) < 0 \). The sign of \( \tilde{\bar{p}}_i'(r) \) is generally unclear since this is the case for \( \tilde{\bar{p}}_i'(r) \).

References


Two-Sided Search in Credit Markets with Imperfect Information

by V. Aivazian
and X. Gu

Figure 1: Identifying the domain of permissible interest rates

Figure 2: The interest effect on project searches by different firms
(The adverse selection and incentive effects)
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Figure 3: The existence of the Bertrand equilibrium rate in two-sided search

Figure 4: The underlying distributions for the bank’s $n$ search models
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Figure 5: The implied distributions for group repayment rates

Figure 6: Identifying all domains of permissible interest rates based on customer grouping
Figure 7: Using $fosd$ to prove the positive dependence of repayments on firm’s type