ECO2704 Lecture Notes: Melitz Model

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• Dynamic Industry Model with heterogeneous firms where opening to trade leads to reallocations of resources within an industry

• Opening to trade leads to
  • Reallocations of resources across firms
  • Low productivity firms exit
  • High productivity firms expand so there is a change in industry composition
  • High productivity firms enter export markets
  • Improvements in aggregate industry productivity
  • No change in firm productivity

• Consistent with empirical evidence from trade liberalizations?
The theoretical model is consistent with a variety of other stylized facts about industries

- Heterogeneous firm productivity
- Ongoing entry and exit
  - Co-movement in (gross) entry and exit due to sunk entry costs
  - Exiting firms are low productivity (selection effect)
- Explains why some firms export within industries and others do not
  - Contrast with traditional theories of comparative advantage
  - Exporting firms are high productivity (selection effect)
  - No feedback from exporting to productivity
• Single factor: labor, numeraire, wage normalized to 1
• Firms enter market by paying sunk entry cost \( (f_e) \)
• Firms observe their productivity \( (\varphi) \) from distribution \( G(\varphi) \)
• Productivity is fixed thereafter
• Once productivity is observed, firms decide whether to produce or exit
• Firms produce horizontally-differentiated varieties, with a fixed production cost \( (f_d) \) and a variable cost that depends on their productivity
• Firms face an exogenous probability of death \( (\delta) \) per period due to *force majeure* events
Preferences and demand

\[ Q = \left[ \int_{k \in \Omega} q(k) \frac{\sigma - 1}{\sigma} dk \right]^{\frac{\sigma}{\sigma - 1}}, \sigma > 1 \]

Let \( R \) be the total expenditures of the representative consumer. Then,

\[ q(k) = \left( \frac{p(k)}{P} \right)^{-\sigma} Q \]

\[ r(k) = p(k)q(k) = \left( \frac{p(k)}{P} \right)^{1-\sigma} R \]

Here \( p(k) \) is the price of variety \( k \) and \( P \) is the price index faced by the consumer,

\[ P = \left[ \int_{k \in \Omega} p(k)^{1-\sigma} dk \right]^{\frac{1}{1-\sigma}} \]
Technology

The amount of labour needed for a firm to produce $q(k)$ units of variety $k$ is

$$f_d + \frac{q(k)}{\varphi(k)}$$

So the total cost of production is

$$f_d + \frac{q(k)}{\varphi(k)}$$

and the marginal cost is

$$\frac{1}{\varphi(k)}$$

All firms with the same productivity behave in exactly the same way. So we will use productivity $\varphi$ rather than variety $k$ to identify a firm.
Profit maximization

Each firm produces a differentiated variety (monopolistic competition):

$$\pi(\varphi) = \max_{q(\varphi)} \{p(\varphi)q(\varphi) - f_d + q(\varphi)/\varphi\}$$

Constant markup pricing:

$$p(\varphi) = \frac{\sigma}{\sigma - 1} \varphi^{-1} = (\rho \varphi)^{-1}$$ (1)

which implies the following:

$$r(\varphi) = (\varphi \rho)^{\sigma - 1} P^{\sigma - 1} R, \quad q(\varphi) = [\varphi \rho]^\sigma P^{\sigma - 1} R$$ (2)

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - f_d$$ (3)
Entry and exit

Prior to entry, a firm incurs a sunk cost of entry, $f_e$. Upon entry, it draws a productivity $\varphi$ randomly from a distribution $G(.)$. The firm will stay in the market only if

$$\pi(\varphi) \geq 0 \quad \text{or} \quad r(\varphi) \geq \sigma f_d$$

Current value of firm $\varphi$ is

$$v(\varphi) = \sum_{t=0}^{\infty} (1 - \delta)^t \max \{\pi(\varphi), 0\} = \max \left\{ \frac{\pi(\varphi)}{\delta}, 0 \right\}$$

Let $\varphi^*$ be the cutoff productivity such that $\pi(\varphi^*) = 0$ and $p_{in}(\varphi^*) = 1 - G(\varphi^*)$ the probability that a firm will stay. Then, productivity distribution of producing firms is given by the following density function:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{p_{in}(\varphi^*)} & \text{if } \varphi \geq \varphi^* \\ 0 & \text{otherwise} \end{cases} \quad (4)$$
Equilibrium in a closed economy

A stationary equilibrium is a vector \((\varphi^*, M, M_e, P, R)\) of cutoff productivity, mass of producing firms, mass of entrants, price level and aggregate expenditures such that price, revenue, quantity and profit for each firm \(\varphi\) are given by equation (1) to (3), the distribution of producing firms is given by \(\mu(\varphi)\) in equation (4), and the following conditions hold:

Zero profit condition:

\[
\pi(\varphi^*) = 0
\]

Free entry condition:

\[
p_{in}(\varphi^*) \int \frac{\pi(\varphi)}{\delta} \mu(\varphi) d\varphi = f_e
\]

Stationarity condition:

\[
p_{in}(\varphi^*) M_e = \delta M
\]

Market clearing condition:

\[
R = L
\]
Price index

\[ P = \left[ \int p(\varphi)^{1-\sigma} \mu(\varphi) Md \varphi \right]^{\frac{1}{1-\sigma}} \]

\[ P = M^{1/(1-\sigma)} \rho^{-1} \left[ \int \varphi^{\sigma-1} \mu(\varphi) d\varphi \right] = M^{1/(1-\sigma)} (\rho \bar{\varphi})^{-1} \quad (5) \]

Here

\[ \bar{\varphi} = \left[ \int \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} \]

is a weighted average of firm productivities.
Profit function and entrants’ expected value

From (2), (3), (5) and the market clearing condition:

\[ \pi(\varphi) = \left( \frac{\varphi}{\bar{\varphi}} \right)^{\sigma-1} \frac{L}{\sigma M} - f_d \]

Entrants’ expected value:

\[ \overline{v} = p_{in}(\varphi^*) \int \frac{\pi(\varphi)}{\delta} \mu(\varphi) d\varphi = p_{in}(\varphi^*) \frac{\pi(\bar{\varphi})}{\delta} = p_{in}(\varphi^*) \frac{L}{\sigma M} - f_d \]
Mass of producing firms

Zero profit condition \[ \Rightarrow \]

\[
\left( \frac{\varphi^*}{\bar{\varphi}} \right)^{\sigma-1} \frac{L}{\sigma M} = f_d
\]

Free entry condition \[ \Rightarrow \]

\[
\frac{p_{in}(\varphi^*)}{\delta} \left[ \frac{L}{\sigma M} - f_d \right] = f_e
\]

Thus,

\[
\left[ \left( \frac{\varphi^*}{\bar{\varphi}} \right)^{1-\sigma} - 1 \right] f_d = \frac{\delta f_e}{p_{in}(\varphi^*)}
\]
Cutoff productivity in equilibrium

\[
\left[ \left( \frac{\varphi^*}{\varphi} \right)^{1-\sigma} - 1 \right] f_d = \frac{\delta f_{e}}{p_{in}(\varphi^*)}
\]

\[\Rightarrow H(\varphi^*) \equiv \int_{\varphi^*}^{\infty} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma^{-1}} - 1 \right] g(\varphi) d\varphi = \frac{\delta f_{e}}{f_d} \quad (7)\]

LHS is a decreasing function, so \(\varphi^*\) increases in fixed production cost \(f_d\), but decreases in entry cost \(f_e\)
Number of firms in equilibrium

From equation (6),

\[ M = \frac{L}{\sigma [f_d + \delta f_e/p_{in}(\varphi^*)]} \]
Size distribution in equilibrium

Firm $\varphi$ employs $z(\varphi) = q(\varphi)/\varphi$ number of worker for production

$$z(\varphi) = \left( \frac{p(\varphi)}{P} \right)^{-\sigma} Q / \varphi = \sigma \rho f_d \left( \frac{\varphi}{\varphi^*} \right)^{\sigma - 1}$$

Thus, the average firm size is

$$\overline{z} = \sigma \rho f_d \left( \frac{\varphi}{\varphi^*} \right)^{\sigma - 1} = \sigma \rho \left[ f_d + \delta_f \rho_p \right]$$
Properties of Closed Economy Equilibrium

- Increasing fixed production cost $f_d$ will
  - raise average productivity
  - raise average firm size
  - but lower the number of producing firms

- Increasing sunk entry cost $f_e$ will
  - lower average productivity
  - lower average firm size
  - and lower the number of producing firms
• $n + 1$ identical countries, so identical wage, price level and income across countries
  • wage is normalized to 1
• To produce, a firm will first incur a fixed production cost $f_d$
• If the firm decides to export, it will also incur a fixed export cost $f_x$
• Iceberg trade cost $\tau$
Trade costs

Same markup pricing for exports:

\[ p_x(\varphi) = \frac{\tau}{\varphi} = \tau p_d(\varphi) \]

Revenue functions:

\[ r_d(\varphi) = (\varphi)^{\sigma-1} P^{\sigma-1} R \quad r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi) \]

Profit functions:

\[ \pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f_d \quad \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x \]

\[ \pi(\varphi) = \pi_d(\varphi) + n\pi_x(\varphi) \]
Cutoff productivities

The cutoff productivity below which a firm will now produce: $\varphi^*_d$

$$\pi_d (\varphi^*_d) = 0$$

Nominal income remains the same as in closed economy: $R = L$
Price level is lower than that in the closed economy

$$\Rightarrow \pi_d (\varphi) < \pi_a (\varphi) \quad \Rightarrow \varphi^*_a < \varphi^*_d$$

Trade increases competition and forces some inefficient firms to exit

The cutoff productivity for exporting: $\max \{ \varphi^*_d, \varphi^*_x \}$

$$\pi_x (\varphi^*_x) = 0$$

If $\tau^{\sigma-1} f_x > f_d$, then $\varphi^*_d < \varphi^*_x$: not all producing firms export, and exporting firms are more productive
Open economy results

- The opening of trade leads to:
  - Rise in the zero profit cutoff productivity
  - Rise in average firm revenue and profit
- Low productivity firms between $\varphi_a^*$ and $\varphi_d^*$ exit
- Intermediate productivity firms between $\varphi_d^*$ and $\varphi_x^*$ contract
- Only firms with productivities greater than $\varphi_x^*$ enter export markets and expand
- All of the above lead to a change in industry composition that raises aggregate industry productivity
Subsequent literature

  - Introduces both exports and FDI as alternative means of serving a foreign market
  - Introduces an outside sector to tractably characterize equilibrium with many asymmetric sectors
  - Combines the Melitz model with the Antras (2003) model of incomplete contracts and trade
  - Incorporates the Melitz model into the framework of integrated equilibrium of Helpman and Krugman (1985)
Subsequent literature

  • Provides a simplified static version of the Melitz model without ongoing firm entry and with an outside sector
  • Examines the model’s implications for the extensive and intensive margins of international trade

  • Solves the Chaney version of the model without an outside sector
  • Derives a sufficient statistic for welfare of the same form as that in Eaton and Kortum (2002)