ECO2704 Lecture Notes: Benjamin Moll's Paper

"Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?"

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- Financial development varies systematically with income across countries
  - Firms in poor countries rely more on internal financing rather than external financing
- Cross-country differences in average marginal product is small (Caselli and Fryer, QJE 2007)
- Dispersion in marginal product of capital is higher in poor countries than in rich countries (Banerjee and Duflo, 2005)
- Real interest rate on savings is low in poor countries

- A static model that is consistent with these facts
- Dynamic model raises question about why capital market distortions do not disappear over time

Static Model

- Continuum of entrepreneurs, indexed by (a, z) (Asset a and productivity z)
  - Joint distribution $g(a, z)$ and marginal distributions $\varphi(a)$ and $\psi(z)$, $a \in [0, \infty)$, $z \in [1, \infty)$
  - Supply of capital is determined by the distribution: $K = \int ag(a, z)dadz$
- Fixed supply of labour L
- Production technology: $y(z) = (zk)^{\alpha} l^{1-\alpha}$
- Financing constraint: $k \leq \lambda a$

Firm optimization problem

$$\pi(a, z) = max_{l,k \leq \lambda a} \left\{ (zk)^{\alpha} l^{1-\alpha} - Rk - wl \right\}$$

For any given k, first solve the labour choice problem:

$$max_l \left\{ (zk)^{\alpha} l^{1-\alpha} - wl \right\}$$

$\Longrightarrow$

$$l = \left(\frac{1-\alpha}{w} \right)^{1/\alpha} zk$$

$$(zk)^{\alpha} l^{1-\alpha} - wl - Rk = \left[\alpha \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)/\alpha} z - R \right] k$$

Factor demand and profits

$$l(a,z) = \left(\frac{1-\alpha}{w}\right)^{1/\alpha} zk(a,z)$$

$$k(a,z) = \begin{cases} \lambda a, & z \geq z^* \\ 0 & z < z^* \end{cases}$$

$$\pi(a,z) = \begin{cases} \left[\alpha \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z - R\right] \lambda a, & z \geq z^* \\ 0 & z < z^* \end{cases}$$

Here

$$z^* = \frac{R}{\alpha}\left(\frac{1-\alpha}{w}\right)^{-(1-\alpha)/\alpha} \tag{1}$$

Market Clearing Conditions

Labor Market:

$$\int l(a,z)g(a,z)dadz = L \tag{2}$$

Capital Market:

$$\int k(a,z)g(a,z)dadz = K \tag{3}$$

Wealth share:

$$\omega(z) = \frac{1}{K}\int ag(a,z)da$$

Then,

Equilibrium cutoff productivity and wage

(2) and (3)$\Longrightarrow$

$$\left(\frac{1-\alpha}{w}\right)^{1/\alpha}\lambda\int_{z^*}^{\infty} z\omega(z)dz = L/K$$

$$\lambda\int_{z^*}^{\infty}\omega(z)dz = 1 \tag{4}$$

The last equation can be used to solve for z^*, which increases in λ
From the first equation, we can solve for equilibrium wage:

$$w = (1-\alpha)ZK^{\alpha}L^{-\alpha} \tag{5}$$

Here

$$Z = \left(\lambda\int_{z^*}^{\infty} z\omega(z)dz\right)^{\alpha} = \left(\frac{\int_{z^*}^{\infty} z\omega(z)dz}{\int_{z^*}^{\infty}\omega(z)dz}\right)^{\alpha} \tag{6}$$

Equilibrium Rental Price and profit

Substituting (5) into (1) $\Longrightarrow$

$$R = \xi \alpha Z K^{\alpha-1} L^{1-\alpha}$$

Here

$$\xi = \frac{z^*}{Z^{1/\alpha}} = \frac{z^* \int_{z^*}^{\infty} \omega(z) dz}{\int_{z^*}^{\infty} z \omega(z) dz} < 1$$

The profit is

$$\pi(a, z) = \pi(z) \lambda a$$

$$\pi(z) = max \left\{ \alpha \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)/\alpha} z - R, 0 \right\} = \nu \alpha Z K^{\alpha-1} L^{1-\alpha}$$

$$\nu = max \left\{ \frac{z - z^*}{Z^{1/\alpha}}, 0 \right\}$$

Equilibrium Output

$$y(a,z) = \begin{cases} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z\lambda a, & z \geq z^* \\ 0 & z < z^* \end{cases}$$

The aggregate GDP is

$$Y = \int y(a,z)g(a,z)dadz = \int_{z^*}^{\infty} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} z\lambda a g(a,z)dadz$$

$$Y = \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)/\alpha} \lambda K \int_{z^*}^{\infty} \omega(z)dz = ZK^{\alpha}L^{1-\alpha}$$

Thus

$$w = (1-\alpha)Y/L \quad , \quad R = \xi\alpha Y/K < \alpha Y/K$$

Example with Pareto productivity distribution

Suppose that a and z are independent and $\psi(z) = \eta z^{-\eta-1}$ $(\eta > 1)$. Then,

$$\omega(z) = \eta z^{-\eta-1}$$

$$\int_{z^*}^{\infty} \omega(z)dz = z^{*-\eta}$$

$$\int_{z^*}^{\infty} z\omega(z)dz = \frac{\eta}{\eta-1} z^{*-\eta+1}$$

And we have

$$z^* = \lambda^{1/\eta}, \quad Z = \left(\tfrac{\eta}{\eta-1}\lambda^{1/\eta}\right)^{\alpha}, \quad \xi = \frac{\eta-1}{\eta}$$

Note that in this case the distribution of a is irrelevant for aggregate TFP and factor prices

Dynamic Model

- A continuum of entrepreneurs, with fixed initial distribution $g_0(a, z)$
  - Fixed initial aggregate capital stock: $K_0 = \int a g_0(a, z) da dz$
- z_t is a Markov process and a_t is endogenously chosen by entrepreneurs for $t > 0$
  - $a_{t+1} = h_t(a_t, z_t)$
- Fixed supply L of workers, who consumes the wage income in every-period
  - no asset accumulation by workers

Equilibrium in a closed economy

Preferences

$$U = E\left[\sum_{t=0}^{\infty} \beta^t log(c_t)\right]$$

Entrepreneur (a_t, z_t)'s decision problem:

$$v_t(a_t, z_t) = max\left\{log(c_t) + \beta E\left[v(a_{t+1}, z_{t+1})|z_t\right]\right\}$$

subject to the constraints

$$c_t + a_{t+1} = B_t(z_t)a_t$$

$$B_t(z_t) = \pi_t(z_t)\lambda + (1 + R_t - \delta)$$

$$\pi_t(z_t) = \begin{cases} \alpha\left(\frac{1-\alpha}{w_t}\right)^{(1-\alpha)/\alpha} z_t - R_t, & z_t \geq z_t^* \\ 0 & z < z_t^* \end{cases}$$

The solution is

$$a_{t+1} = \beta B_t(z_t)a_t$$

Evolution of aggregate capital stock

$$K_{t+1} = \beta \int B(z_t) a_t g_t(a_t, z_t) da_t dz_t = \beta \left[\int B(z_t) \omega_t(z_t) dz_t \right] K_t$$

Note that ,

$$\pi_t(z_t) = max \left\{ \alpha \left(\frac{1-\alpha}{w_t} \right)^{(1-\alpha)/\alpha} z_t - R_t, 0 \right\} = \nu_t \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}$$

$$\nu_t = max \left\{ \frac{z_t - z_t^*}{Z_t^{1/\alpha}}, 0 \right\}$$

Therefore

$$K_{t+1} = \beta \left(1 - \delta + \theta_t \alpha Y_t / K_t \right) K_t$$

Here

$$\theta_t = \xi_t + \lambda \int_{z_t^*}^{\infty} \left(\frac{z_t - z_t^*}{Z_t^{1/\alpha}} \right) \omega_t(z_t) dz_t$$

Using the definition of ξ_t and z_t^* it can be shown that

$$\theta_t = \xi_t + \lambda \int_{z_t^*}^{\infty} \left(\frac{z_t - z_t^*}{Z_t^{1/\alpha}} \right) \omega_t(z_t) dz_t = 1$$

$\Longrightarrow$

$$K_{t+1} = \beta \alpha Y_t + \beta(1-\delta) K_t$$

So, given the aggregate TFP Z_t, the aggregate behavior of this model is the same as that of a standard neoclassical growth model with one exception:

- the real interest rate is lower due to the frictions in the financial market

Except for the real interest rate, financial frictions affect the aggregate economy through the aggregate TFP only

i.i.d. productivity shocks

Assume that a_0 and z_0 are independent and that $\{z_t\}_{t\geq 0}$ is an i.i.d. sequence. Then, $a_{t+1} = h(a_t, z_t)$ is independent of z_{t+1}.
Assume that z_t has the Pareto distribution. Then

$$z_t^* = \lambda^{1/\eta}, \quad Z_t = \left(\frac{\eta}{\eta-1}\lambda^{1/\eta}\right)^{\alpha}, \quad \xi_t = \frac{\eta - 1}{\eta}$$

Parameter η determines how thick the left tail of the distribution is. The lower the value of η

- the thicker the left tail (more low productivity entrepreneurs) and
- the larger the wedge between the real interest rate and the marginal product of capital

If $\eta = 1.2$, the real rental price of capital is only one sixth of the marginal product of capital

Permanent productivity shocks

Assume that $z_t = z_0 = z$ for all t
Since

$$a_{t+1} = \beta B_t(z) a_t$$

and B_t is increasing function in z. The entrepreneur with the highest productivity $\overline{z}$ will accumulate asset fast than everyone else:

$$lim_{t \to \infty} \omega_t(\overline{z}) = 1$$

So, in the limit, the entrepreneur with the highest productivity owns all the asset and the financing constraint does not matter anymore
In the data, the productivity shock appears to be persistent:

- but distortion in capital allocations has been quite persistent
- why distortion has not been undone by asset accumulation of high productivity entrepreneurs?

Related Research

Angeletos (2007) “Uninsured Idiosyncratic Investment Risk and Aggregate Saving” Review of Economic Dynamics

- Analyze a model that is similar to the model discussed here

Banerjee and Moll (2009) “Why Does Misallocation Persist?” forthcoming, American Economic Journal: Macroeconomics

- You can guess what the paper is about from the title

Buera, Kaboski and Shin (2009) "Finance and Development: A Tale of Two Sectors," forthcoming, American Economic Review

- Introduce fixed cost and decreasing returns in production.
- Two sectors, manufacturing (large fixed cost) and services (small fixed cost)
- Because of fixed cost, entrepreneurs with high manufacturing productivity and low wealth may choose to be a worker or entrepreneur in service sector
- Distortion at intensive and extensive margin

Related Research

Song, Storesletten and Zilibotti (2009) "Growing Like China" forthcoming, American Economic Review

- Two sectors: financially constrained and non constrained sectors, but no heterogeneity within each sector
- OLG rather than infinite horizon model
- Explain the high growth, high return to investment and high current account surplus phenomenon of China

Beura and Shin (2010) "Productivity Growth and Capital Flows: The Dynamics of Reform" working paper

- Simplified one-sector version of Buera, Kaboski and Shin (2009)
- Studying the dynamics of saving and investment after elimination of idiosyncratic taxes on output

Aoki, Benigno, and Kiyotaki (2009): "Adjusting to Capital Account Liberalisation" working paper

- Analyze the impact of capital account liberalization in the presence of domestic financial frictions