5 Empirical Implications of the Neoclassical Growth Model

In this section, we examine the ability of the neoclassical growth model in accounting for international income level differences. We focus on level rather than growth rate because the facts on growth rate differences are less clear and less robust. The facts on level differences, however, are clear and robust. In short, the ratio of output per worker in the richest 5 countries to that in the poorest 5 countries has consistently been around 32 during the period 1960-1996. The question we want to ask is: Can the neoclassical growth model account for the income level difference of this large magnitude?

Let's setup a standard one-sector neoclassical growth model.

5.1 The Model

The general setup is as follows: A representative household with N members. The household purchases investment good from producers to accumulate capital, and purchases consumption good from producers to consume. Capital is owned by the household. The consumption good producer rents capital and hires labor from the household. The investment good producer purchases consumption good as production input from the final good producer.

Preferences:

$$\sum_{t=0}^{\infty} \frac{1}{1-\gamma} \beta^t N_t \left[(C_t/N_t)^{1-\gamma} - 1 \right]$$

Here N_t is the size of the representative household in period t, C_t is the household's total consumption. So, the household's utility is the sum of all N_t members utilities, which depend on per capita consumption, C_t/N_t.

Technologies:

Two types of goods are produced in this economy: consumption good and investment good, both are produced with standard Cobb-Douglas technologies with labor and capital as inputs:

$$\begin{aligned} Y_{Ct} &= AK_{Ct}^{1-\alpha}(h_t N_{Ct})^{\alpha} \\ Y_{It} &= \pi^{-1} AK_{It}^{1-\alpha}(h_t N_{It})^{\alpha} \end{aligned}$$

Here π is a parameter that represents the relative efficiency of the consumption good production relative to investment good production.

Representative household's problem:

$$\max_{\{C_t, K_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \frac{1}{1-\gamma} \beta^t N_t \left[(C_t/N_t)^{1-\gamma} - 1 \right]$$

subject to

$$C_t + q_t(I_t - \theta\delta K_t) = (1-\theta) r_t K_t + w_t N_t + T_t$$
$$K_{t+1} = (1-\delta) K_t + I_t$$

and

$$C_t \geq 0, I_t \geq 0.$$

The price of consumption good is normalized to one, q_t is the price of capital in units of consumption, or relative price of capital to consumption, r_t the rental price of capital, w_t the price of labor service. The government levies a flat-rate tax on capital income. The tax rate is θ. Capital depreciation is deductible from capital income. So, the net investment expenditure is $q_t(I_t - \theta\delta K_t)$. The tax revenues are rebated to households as lump-sum transfers. So,

$$T_t = \theta r_t \overline{K}_t$$

Here $\overline{K}_t$ is the aggregate capital stock in the economy.

Firms' maximization problems:

For the consumption good producing firm,

$$\max_{K_{Ct}, N_{Ct}} \left[Y_{Ct} - r_t K_{Ct} - w_t N_{Ct} \right]$$

which implies that

$$r_t = (1-\alpha) A \left(\frac{K_{Ct}}{h_t N_{Ct}} \right)^{-\alpha} \quad (10)$$

$$w_t = \alpha A \left(\frac{K_{Ct}}{h_t N_{Ct}} \right)^{1-\alpha} h_t \quad (11)$$

And for the investment good producing firm:

$$\max_{K_{It},N_{It}} [q_t Y_{It} - r_t K_{It} - w_t N_{It}]$$

which implies that

$$r_t = (1-\alpha) q_t \pi^{-1} A \left(\frac{K_{Ct}}{h_t N_{Ct}}\right)^{-\alpha} \quad (12)$$

$$w_t = \alpha q_t \pi^{-1} A \left(\frac{K_{Ct}}{h_t N_{Ct}}\right)^{1-\alpha} h_t \quad (13)$$

From equation (10) to (13) we have

$$\frac{K_{Ct}}{N_{Ct}} = \frac{K_{Ct}}{N_{Ct}} = \frac{(1-\alpha)w_t}{\alpha r_t}.$$

Since the capital-labor ratio is the same across the two sectors, it must be the case that, in equilibrium,

$$\frac{K_{Ct}}{N_{Ct}} = \frac{K_{Ct}}{N_{Ct}} = \frac{K_t}{N_t}.$$

From (10) and (12) we have

$$(1-\alpha)A\left(\frac{K_{Ct}}{h_t N_{Ct}}\right)^{-\alpha} = (1-\alpha) q_t \pi^{-1} A \left(\frac{K_{Ct}}{h_t N_{Ct}}\right)^{-\alpha}.$$

Since $\frac{K_{Ct}}{N_{Ct}} = \frac{K_{Ct}}{N_{Ct}}$, the above equation implies

$$q_t = \pi.$$

We assume that both the size of labor force, N_t, and the labor productivity, h_t, grow at constant rates:

$$N_t = (1+n)^t, \text{ and } h_t = (1+g)^t.$$

Let

$$c_t = \frac{C_t}{h_t N_t}, i_t = \frac{I_t}{h_t N_t}, k_{Ct} = \frac{K_{Ct}}{h_t N_t}, k_{It} = \frac{K_{It}}{h_t N_{It}}, \overline{k}_t = \frac{\overline{K}_t}{h_t N_t}, \overline{k}_t = \frac{\overline{K}_t}{h_t N_t},$$

$$r(\overline{k}) = (1-\alpha)A\overline{k}^{-\alpha}, \ w(\overline{k}) = \alpha A \overline{k}^{1-\alpha}, \text{ and } \tau(\overline{k}) = \theta[r(\overline{k}) - \pi\delta]\overline{k}.$$

Assume that the aggregate investment (normalized) is also a function of the normalized aggregate capital stock, $\overline{k}_t$: $\overline{i}_t = \varphi(\overline{k}_t)$.

Then, the household's problem can be rewritten as follows:

$$\max_{\{c_t, i_t, k_{t+1}\}_{t\geq 0}} \sum_{t=0}^{\infty} \frac{1}{1-\gamma} \widehat{\beta}^t (c_t^{1-\gamma} - 1)$$

subject to[6]

$$c_t + \pi(i_t - \theta\delta k_t) = (1-\theta) r(\overline{k}_t) k_t + w(\overline{k}_t) + \tau(\overline{k}_t),$$

$$\begin{aligned} k_{t+1}(1+\xi) &= (1-\delta)k_t + i_t \\ \overline{k}_{t+1}(1+\xi) &= (1-\delta)\overline{k}_t + \varphi(\overline{k}_t) \end{aligned}$$

$$c_t \geq 0,\ k_{t+1} \geq 0.$$

Here,

$$\widehat{\beta} = \beta(1+\xi)/(1+g)^{\gamma}.$$

Substituting the equation for i_t in to the budget constraint yields

$$c_t + \pi(1+\xi)k_{t+1} = R(\overline{k}_t)k_t + w(\overline{k}_t) + \tau(\overline{k}_t).$$

Here,

$$R(\overline{k}) = \pi + (1-\theta)(r(\overline{k}) - \pi\delta).$$

We can rewrite the household's problem as a dynamic programming problem:

$$v(k, \overline{k}) = \max_{k'} \left\{ \frac{1}{1-\gamma} \left(\left[R(\overline{k})k + w(\overline{k}) + \tau(\overline{k}) - \pi(1+\xi)k' \right]^{1-\gamma} - 1 \right) + \widehat{\beta} v(k', \overline{k}') \right\}$$

subject to

$$0 \leq k' \leq [\pi(1+\xi)]^{-1} \left[R(\overline{k})k + w(\overline{k}) + \tau(\overline{k}) \right]$$

and

$$\overline{k}' = (1+\xi)^{-1}[(1-\delta)\overline{k} + \varphi(\overline{k})].$$

[6] Note that the utility function above differs from the original one by a constant term: $\sum_{t=0}^{\infty} \frac{1}{1-\gamma} \beta^t (h_t^{1-\gamma} - 1)$. So the optimization problem under this utility function is the same as the original problem.

Definition *A* ***recursive competitive equilibrium*** *consists of price functions* $(q(.), r(.), w(.))$*, government policy rule* $(\theta, \tau(.))$*, household's decision rule* $k'(.,.)$*, firms' decision rule* $k_f(.)$ *and aggregate investment function* $\varphi(.)$ *such that*

(1) given price functions and policy rule, $k'(.,.)$ *solves the household's dynamic programming problem, and the firms' decision rule are such that*

$$\begin{aligned} r(\overline{k}) &= (1-\alpha)A\left(k_f(\overline{k})\right)^{-\alpha}, \\ w(\overline{k}) &= \alpha A\left(k_f(\overline{k})\right)^{1-\alpha}; \end{aligned}$$

(2) $q(\overline{k}) = \pi$*;*
(3) markets clear

$$\overline{k} = k = k_f(\overline{k}), \text{ and } \varphi(\overline{k}) = (1+\xi)k'(k,\overline{k}) - (1-\delta)k.$$

The first order condition of the household's maximization problem is:

$$c^{-\gamma}\pi(1+\xi) = \widehat{\beta}v_1(k', \overline{k}')$$

and the envelope condition implies that

$$v_1(k, \overline{k}) = c^{-\gamma}R(\overline{k})$$

Combining the two yields the following Euler equation:

$$c^{-\gamma}\pi(1+\xi) = \widehat{\beta}c'^{-\gamma}R(\overline{k}')$$

Since cross country income level differences are very persistent, we need to look for long-term rather than transitional factors. One natural way to examine the long-term factors is to focus on the steady-state income.

In a steady state, $c = c'$ and $\overline{k} = \overline{k}'$. So, the Euler equation becomes

$$\pi(1+\xi) = \widehat{\beta}R(\overline{k})$$

which yields the following solution for $\overline{k}$,

$$\overline{k} = \left\{ \frac{(1-\alpha)(1-\theta)A}{\pi\left[\beta^{-1}(1+g)^{\gamma} - 1 + (1-\theta)\delta\right]} \right\}^{1/\alpha}.$$

Thus, the output per worker is

$$\begin{aligned} Y_t/N_t &= A\overline{k}^{1-\alpha}(1+g)^t \\ &= A^{\frac{1}{\alpha}}\left(\frac{1}{\pi}\right)^{\frac{1-\alpha}{\alpha}}\left[\frac{(1-\alpha)(1-\theta)}{\beta^{-1}(1+g)^{\gamma}-1+(1-\theta)\delta}\right]^{\frac{1-\alpha}{\alpha}}(1+g)^t \end{aligned}$$

So, three variables affect the economy's labor productivity in the steady-state: TFP A, investment efficiency $1/\pi$, and the tax rate on capital income θ. If we have data on these variables, then we can use the equation above to see how much of the income differences can be accounted for by each of these variables.

In the steady state, we have

$$i = (\xi + \delta)\overline{k},$$

so, the investment rate in domestic price is

$$\text{invrate}_d = \frac{\pi i(1+g)^t}{Y_t/N_t} = \frac{\pi(\xi+\delta)\overline{k}}{A\overline{k}^{\alpha}}.$$

Substituting the equation for $\overline{k}$ into the equation above yields

$$\text{invrate}_d = \left[\frac{(1-\alpha)(1-\theta)}{\beta^{-1}(1+g)^{\gamma}-1+(1-\theta)\delta}\right](\xi+\delta)$$

which is independent of the TFP and the investment efficiency variable. The investment rate in international prices, however, is inversely related to investment efficiency:

$$\text{investment rate in international price} = \frac{\pi^*}{\pi}\text{invrate}_d$$

But it is still independent of the TFP variable. We can now write the output per worker as follows:

$$\text{output per worker} = [TFP]^{\frac{1}{1-\alpha}}\left(\frac{1}{\pi}\right)^{\frac{1-\alpha}{\alpha}}[\text{invrate}_d]^{\frac{1-\alpha}{\alpha}}(\xi+\delta)^{-\frac{1-\alpha}{\alpha}}(1+g)^t.$$

Thus, we can write the percentage difference in output per worker between the richest and poorest countries as

$$\ln\left(\frac{\text{output per worker}_{rich}}{\text{output per worker}_{poor}}\right)$$
$$= \frac{1}{1-\alpha}\ln\left(\frac{TFP_{rich}}{TFP_{poor}}\right) + \frac{1-\alpha}{\alpha}\left[\ln\left(\frac{\pi_{poor}}{\pi_{rich}}\right) + \ln\left(\frac{\text{invrate}_{drich}}{\text{invrate}_{dpoor}}\right) + \ln\left(\frac{(\xi+\delta)_{poor}}{(\xi+\delta)_{rich}}\right)\right]$$

5.2 Facts

Investment rate in domestic price:

In PWT data set, this rate does not vary systematically with income across countries. Therefore, distortions in capital market that depress savings cannot be an important factor in accounting for income differences.

Barriers to Investment:

In equilibrium, the relative price of capital $q_t = \pi$. Thus, we can use the relative price of capital as a measure of barriers to investment. In PWT data set, the top 5% countries' relative price of capital is one-fourth of that in the bottom 5% countries. That is,

$$\frac{\pi_{poor}}{\pi_{rich}} = 4.$$

According to the model, this would imply that the ratio of investment rate in international prices of 4:1 between the richest 5% and poorest 5% countries. Restuccia and Urrutia (2001) find that this implication is consistent with the data. More generally, they found that the change in relative prices over time also help explain the chang in the investment rate over time.

As pointed out by Hsieh and Klenow (2007), the difference in relative price of capital reflects a difference in the efficiency in producing investment/capital goods. In other words, the poor countries have relatively low investment rate because they cannot produce capital good efficiently.

How much the low investment rate can account for the difference in output per worker depends on the capital share $1-\alpha$. If we use the standard value of 1/3 for capital share, then, the contribution of the barriers to investment on income disparity is

$$\ln\left(4^{\frac{1/3}{2/3}}\right) / \ln(32) = 20\%.$$

Even if we use a larger capital share of 0.5, the differences in the barriers to investment can still only account for 40% of the differences.

So far, we have concentrated on the steady state implication of the neoclassical growth model because the income level differences are very persistent. Some may argue that there may be transitory factors that are important in accounting for the observed differences. In the next section, we do an accounting exercise that does not rely on the steady state assumption.

5.3 Development Accounting

The only assumption we will make here is that the output is produced using a Cobb-Douglas production function.

$$\begin{aligned} Y &= AK^{\alpha}(hN)^{1-\alpha} \\ y &= Ak^{\alpha}h^{1-\alpha} \end{aligned}$$

Here, $y = Y/N$ and $k = K/N$. To do an accounting exercise, we need to have data on K, h, and N. The data on population or labor force is readily available in the PWT data set. Both physical and human capital stocks, however, have to be estimated.

Using data on investment, I_t, and the capital accumulation equation to estimate K:

$$K_t = (1-\delta)K_{t-1} + I_t,\ \delta = 0.06.$$

Using wage and schooling data to estimate h:

In the theory section, we have shown that the equilibrium wage is of the form: $w(\overline{k})h_t$, where $\overline{k}$ is a variable that is independent of the human capital. Thus, we can write wage as a linear function of human capital level,

$$w = \overline{w}h$$

Next, we assume that the human capital stock is a function of average years of schooling, s:

$$h = h(s).$$

Then wage is a function of years of schooling.

$$w(s) = \overline{w}h(s)$$

Starting from the seminal work of Mincer, many people have estimated wage-schooling equation (often called Mincer regression) as follows:

$$\ln w = \text{constant} + \phi(s)$$

After the function $\phi(s)$ is estimated, we can calculate the human capital stock as

$$h = e^{\phi(s)}.$$

Francesco Caselli estimated k and h based on the data in PWT 6.0 using the methods above. He then calculated the value of the following variable for each country in the data set:

$$\widehat{y} = \widehat{k}^{\alpha}\widehat{h}^{1-\alpha}.$$

If all countries' production can be characterized by Cobb-Douglas production function and if they all have the same TFP, then $\widehat{y}$ should be very close to the actual data, y. If $\widehat{y}$ is different from y, then, the Cobb-Douglas production function cannot fully account for the observed income differences. Caselli used two measures to guage the sucess of the Cobb-Douglas production function in accouting for international income differences.

$$\begin{aligned}
\text{measure 1} &= \frac{Var(\ln \widehat{y})}{Var(\ln y)} \\
\text{measure 2} &= \frac{\widehat{y}^{90}/\widehat{y}^{10}}{y^{90}/y^{10}}
\end{aligned}$$

where y^n $(\widehat{y}^n)$ means the nth percentile of the distribution of y $(\widehat{y})$. Using the data for 1996, Caselli found that measure 1= 0.40 and measure 2= 0.35. (He assumes that $\alpha = 1/3$.) In other words, differences in human and physical capital can account for only 40% of the variance in output per worker and about one third of the income disparity between the top and bottom 10% countries.

In summary, the neoclassical growth model, which emphasizes differences in capital (physical and/or human), leaves a large portion of the observed income difference unexplained. In other words, a large portion of the observed income differences must be due to differences in production efficiency. Furthermore, even the part where the model explains the data well, the difference in investment rate across countries, also implies that there are large efficiency difference in capital good production.

5.4 One Source of Aggregate Inefficiency: Misallocation across sectors

Consider an economy with two sectors: agriculture and non-agriculture, indexed by a and n, respectively. Assume that only labour is used for production:

$$\begin{aligned} Y_a &= A_a L_a \\ Y_n &= A_n L_n \end{aligned}$$

Here A_i is the productivity in sector i and L_i the employment in sector i ($i = a, n$). Let $L = L_a + L_n$ be the total employment in the economy. Then, the aggregate GDP in the economy is

$$Y = A_a L_a + A_n L_n$$

and the aggregate labour productivity is

$$A = \frac{Y}{L} = A_a \frac{L_a}{L} + A_n \frac{L_n}{L}.$$

Let $l_a = L_a/L$ be the employment share of agriculture, then, we have

$$A = A_a l_a + A_n(1 - l_a).$$

Clearly, an increase in the productivity of either agricultural or non-agricultural sector will result in an increase in aggregate productivity. There is also another channel through which aggregate productivity can be increased: reallocation of labour from agricutlure to non-agriculture. To see this, let's rewrite the equation for the aggregate productivity as follows:

$$A = A_n - (A_n - A_a)l_a.$$

For most economies, productivity in agriculture is less than the productivity in non-agriculture. That is, $A_n - A_a > 0$. Thus, a reduction in the share of employment in agriculture, l_a, will result in an increase in aggregate productivity.

5.4.1 Facts about sectoral productivities

The Food and Agricultural Organization of the United Nations (FAO) has compiled PPP adjusted cross-country data on agrciultural GDP using the same methodology as the one used for Penn World Table (PWT) in constructing the aggregate GDP data. The most recent FAO data available is 1985. Combining the PWT and FAO dataset one can calculate labour productivies by sectors for different countries. I will focuse here on the productivities of the richest 5% and poorest 5% of the countries in the world. Using superscript r for the richest countries and p for the poorest countries. Here are the results:

- The ratio of the aggregate labour productivity in the richest coutries to that in the poorest countries is 32: $A^r/A^p = 32$.
- The ratio of the non-agricultural labour productivity in the richest countries to that in the poorest countries is 3.8: $A_n^r/A_n^p = 3.8$.
- The ratio of the agricultural labour productivity in the richest countries to that in the poorest countries is 70.7: $A_n^r/A_n^p = 70.7$.
- The agriculture's share of employment is 5% in the richest countries and 90% in the poorest countries: $l_a^r = 0.05$ and $l_a^p = 0.90$.
- The ratio of agricultural labour productivity to non-agricultural productivity in the richest countries is 0.39: $A_a^r/A_n^r = 0.39$.

We can normalize the non-agricultural labour productivity in the richest countries to one hundred. That is, $A_n^r = 100$. Then, based on the facts listed above, we have

$$\begin{aligned} A_a^r &= 39, \\ A_a^p &= 0.55, \\ A_n^r &= 100, \\ A_n^p &= 26. \\ A^r &= 97 \\ A^p &= 3 \end{aligned}$$

Note that there is an enormous productivity gap between agriculture and non-agriculture within the pooreset countries: $A_a^p/A_n^p = 0.55/26 = 0.02$. While poor countries have low productivity in both sectors, they are particularly unproductive in agriculture. Unfortuantely, most of the employment in these poor countries are in the unproductive agriculture (90%). Moving labour away from agriculture, then, is a potential source of aggregate productivity increase.

5.4.2 Accounting for the difference in aggregate productivity

Based on the data presented in the previous subsection, we can calculate the contribution to aggregate productivity differences from each of the three factors: productvity differences in the two sectors and the difference in labour allocation across the two sectors.

Labour productivity difference in the non-agricultural sector

Suppose that the non-agricultural labour productivity in the poorest countries is the same as that in the richest countries, that is $A_n^p = A_n^r = 100$. Then,

$$\begin{aligned} A^p &= 100 \times 0.1 + 0.55 \times 0.9 = 10.5, \\ A^r/A^p &= 9.2. \end{aligned}$$

That is, by eliminating the cross-country productivity difference in the non-agriculture sector, the aggregate productivity gap will be a factor of 9 rather than 32.

Labour productivity difference in the agricultural sector

In the data, there is a larger gap in labour productivity across sectors. Now suppose that the between sector labour productivity gap in the poor countries is the same as that in the rich countries. That is, $A_a^p/A_n^p = A_a^r/A_n^r = 0.39$. Then,

$$\begin{aligned} A_a^p &= 0.39A_a^n = 0.39 \times 26 = 10.14, \\ A^p &= 26 \times 0.1 + 10.14 \times 0.9 = 11.726, \\ A^r/A^p &= 8.3. \end{aligned}$$

That is, by closing the cross-sector labour productivity gap within the poorest countries can also result in a significant improvment in the aggregate productivity in these countries.

Difference in labour allocation across sectors

As we noted earlier, the poorest countries have an enormous productivity gap between agriculture and the non-agricultural sector. Yes, they allocate 90% of labour in the unproductive agricultural sector. Suppose that these countries reallocate 85% of the labour away from agriculture so that the agriculture's employment share is 5%, the same as that in the richest countries. Then, we have

$$\begin{aligned} A^p &= 26 \times 0.95 + 0.55 \times 0.05 = 24.7, \\ A^r/A^p &= 3.9. \end{aligned}$$

The result shows that the reallocation of labour away from agriculture can result in a very large improvement in the aggregate productivity for the poorest countries, much larger than closing sectoral productivity gaps. This raises an important question: why do poor countries keep so many workers in agriculture?

5.4.3 Why do poor countries keep so many workers in agriculture?

The agricultural sector produces a nesessary good, food. In the absence of trade, a country cannot simply move workers away from agriculture when productivity in the sector is low because by doing that the country may not have enough food to feed the entire population. In fact, if feeding the entire population is a hard constraint, then the lower the productivity in the agricultural sector, the more labour will be needed in the sector to satisfy the food demand. This suggests that increasing labor productivity in agriculture has an additional effect on aggregate productivity through the reduction of share of labour in agriculture.

One way to measure this indirect effect of the improvement in agricultural productivity is by assuming that the poor countries currently produce just enough food to feed the population. That is the total amount of agricultural output equals the basic amount of food consumption each individual needs, call it $\bar{c}$, times the size of the population. Without loss of generality, we also assume that the labour participation rate is one, that is the size of the population equlas the size of the labour force. Then, we have Let's assume that the

$$A_a^p L_a^p = \bar{c} L^p, \tag{14}$$

which implies that

$$l_a^p = \frac{L_a^p}{L^p} = \frac{\overline{c}}{A_a^p}. \tag{15}$$

That is, the share of labour in agriculture is inversely related to the labour productivity in agriculture. Therefore, an increase in labour productivity in agriculture will contributes to the increase in aggregate productivity directly and indirectly through a reduction in the share of labour in agricutlure.

Using this simple model, we can re-examine the impact of closing the productivity gap between the agricultural and the non-agricultural sector. Equation (14) also implies that not only leads to an increase in the labour productivity leads to a reduction in

$$\overline{c} = A_a^p L_a^p / L^p = A_a^p l_a^p.$$

From the data presented at the beginning of this section, we have

$$\overline{c} = 0.55 \times 0.9 = 0.495.$$

If now we suppose that the between sector labour productivity gap in the poor countries is the same as that in the rich countries. That is, $A_a^p/A_n^p = A_a^r/A_n^r = 0.39$. Then,

$$\begin{aligned} A_a^p &= 0.39 A_a^n = 0.39 \times 26 = 10.14, \\ l_a^p &= \overline{c}/A_a^p = 0.495/10.14 = 0.0488, \\ A^p &= 26 \times 0.9522 + 10.14 \times 0.0488 = 25.23, \\ A^r/A^p &= 3.85. \end{aligned}$$

In other words, if we take into account the indirect effect of the increase in agricultural productivity on labour alloaction across sectors, closing the between-sector labour productivity gap has a much larger impact on the aggregate labour productivity then the case when labour allocation is fixed independently.