6 Structural Change

Over the last one hundred years or so, there are two sets of stylized facts that characterize the US economy.

Kaldor facts: the growth rate of per capita GDP, the capital to output ratio, the real interest rate, and the share of labor income in national income have all stayed at roughly constant levels.

Kuznets facts: as the economy grows, the output and employment share of agricutlure declines and the corresponding share of services increases.

We know that the simple one-sector neoclassical growth model is consistent with the Kaldor facts. In this section we study a multi-sector growth model and study the conditions under which both the Kaldor facts and Kuzents facts are consistent with the model.

Consider an economy with a representative household whose preferences are given as follows:

Preferences

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left[c_t^{1-\gamma} - 1\right].$$

Here, c_t is a composite of m differentiated consumption goods,

$$c_t = \left[\sum_{i=1}^{m} \phi_i (c_{it} - b_i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

and ε is the elasticity of substitution and b_i is a constant, for $i = 1, ..., m$.

Technology

The technology for producing good i is

$$Y_{it} = A_{it} K_{it}^{1-\alpha_i} L_{it}^{\alpha_i}, \ 0 < \alpha_i < 1.$$

For $i = 1, ..., m-1$, the output of sector i can only be used for consumption. For sector m, however, the output can also be used for investment in capital that is used for production in all sectors.

To solve the equilibrium, we first look at the household's dynamic programming problem:

$$v(K_t) = \max \left\{ \frac{1}{1-\gamma} \left[c_t^{1-\gamma} - 1\right] + \beta v(K_{t+1}) \right\}$$

subject to

$$c_t = \left[\sum_{i=1}^{m} \phi_i (c_{it} - b_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$\sum_{i=1}^{m} p_{it} c_{it} + X_t = w_t L_t + r_t K_t,$

$$K_{t+1} = (1-\delta) K_t + X_t.$$

The problem can be solved in two steps. First, we solve the optimal consumption allocation across the m goods given X_t and $w_t L + r_t K_t$, which is a static optimization problem. Then, we solve for the optimal X_t in the dynamic programing problem.

The consumption allocation problem:

$$\max_{c_{it}, i=1,\ldots,m} \left[\sum_{i=1}^{m} \phi_i (c_{it} - b_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

subject to

$$\sum_{i=1}^{m} p_{it} c_i = w_t L + r_t K_t - X_t.$$

The first order conditions are:

$$\lambda p_{it} = \left[\sum_{i=1}^{m} \phi_i (c_{it} - b_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}-1} \phi_i (c_{it} - b_i)^{\frac{\varepsilon-1}{\varepsilon}-1}$$

or

$$\begin{aligned} c_{it} &= \left[\sum_{i=1}^{m} \phi_i (c_{it} - b_i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \lambda^{-\varepsilon} \phi_{it}^{\varepsilon} p_{it}^{-\varepsilon} + b_i \\ &= c_t \lambda^{-\varepsilon} \phi_{it}^{\varepsilon} p_{it}^{-\varepsilon} + b_i. \end{aligned}$$

Substituting it into the budget constraint, we have

$$c_t \lambda^{-\varepsilon} \sum_{j=1}^{m} \phi_j^{\varepsilon} p_{jt}^{1-\varepsilon} + \sum_{j=1}^{m} p_{jt} b_j = w_t L + r_t K_t - X_t.$$

Thus, $c_t \lambda^{-\varepsilon} = \frac{1}{\sum_{j=1}^{m} \phi_j^{\varepsilon} p_{jt}^{1-\varepsilon}} w_t L + r_t K_t - X_t - \sum_{j=1}^{m} p_{jt} b$

$$c_{it} = \frac{\phi_i^{\varepsilon} p_{it}^{-\varepsilon}}{\sum_{j=1}^{m} \phi_j^{\varepsilon} p_j^{1-\varepsilon}} \left(w_t L + r_t K_t - X_t - \sum_{j=1}^{m} p_{jt} b_j \right) + b_i, \qquad (16)$$

$$
\begin{aligned}
c_t &= \left[\sum_{i=1}^{m}\phi_i(c_{it}-b_i)^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad (17)\\
&= \left(\sum_{i=1}^{m}\phi_i^{\varepsilon}p_{it}^{1-\varepsilon}\right)^{-\frac{1}{1-\varepsilon}}\left(w_tL+r_tK_t-X_t-\sum_{j=1}^{m}p_{jt}b_j\right)\\
&= p_t^{-1}\left(w_tL+r_tK_t-X_t-\sum_{j=1}^{m}p_{jt}b_j\right),
\end{aligned}
$$

where

$$p_t=\left(\sum_{i=1}^{m}\phi_i^{\varepsilon}p_{it}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

is the consumer-price-index (CPI).

Thus, the household's dynamic programming problem becomes

$$v(K_t)=\max_{K_{t+1}}\left\{\frac{1}{1-\gamma}\left[p_t^{-(1-\gamma)}\left(w_tL+r_tK_t+(1-\delta)K_t-K_{t+1}-\sum_{j=1}^{m}p_{jt}b_j\right)^{1-\gamma}-1\right]+\beta v(K_{t+1})\right\}$$

The first-order-condition is

$$p_t^{-(1-\gamma)}c_t^{-\gamma}=\beta v(K_{t+1})$$

and the envelope condition is

$$v'(K_t)=p_t^{-(1-\gamma)}c_t^{-\gamma}(1+r_t-\delta).$$

Thus, we have

$$p_t^{-(1-\gamma)}c_t^{-\gamma}=\beta p_{t+1}^{-(1-\gamma)}c_{t+1}^{-\gamma}(1+r_{t+1}-\delta).$$

Firms' problem:

For each $i=1,...,m$, the firm in sector i chooses capital and labor to maximize profits:

$$\max_{K_{it},L_{it}}\{p_{it}A_{it}K_{it}^{1-\alpha_i}L_{it}^{\alpha_i}-w_tL_{it}-r_tK_{it}\}$$

The first order conditions of the optimization problem are:

$$\begin{aligned} \alpha_i p_{it} A_{it} \left(\frac{K_{it}}{L_{it}}\right)^{1-\alpha_i} &= w_t, \\ (1-\alpha) p_{it} A_{it} \left(\frac{K_{it}}{L_{it}}\right)^{-\alpha_i} &= r_t. \end{aligned}$$

These two conditions imply that

$$\frac{K_{it}}{L_{it}} = \frac{1-\alpha_i}{\alpha_i}\frac{w_t}{r_t}, \;\; i = 1, ..., m.$$

Let $k_{it} = K_{it}/L_{it}$, then, the equation above implies that

$$k_{it} = \frac{(1-\alpha_i)\alpha_m}{\alpha_i(1-\alpha_m)} k_{mt} = \mu_i k_{mt} \tag{18}$$

where

$$\mu_i = \frac{(1-\alpha_i)\alpha_m}{\alpha_i(1-\alpha_m)}.$$

Thus, we have

$$\begin{aligned} w_t &= \alpha_m A_{mt} k_{mt}^{1-\alpha_m} = p_{it}\alpha_i \mu_i^{\alpha_i} A_{it} k_{mt}^{1-\alpha_i}, \; i = 1, ..., m-1. && (19) \\ r_t &= (1-\alpha_m) A_{mt} k_{mt}^{-\alpha_m}. && (20) \end{aligned}$$

The first equation above implies that

$$p_{it} = \frac{\alpha_m}{\alpha_i \mu_i^{\alpha_i}} \frac{A_{mt}}{A_{it}} k_{mt}^{\alpha_i - \alpha_m}. \tag{21}$$

That is, the relative price of good i is inversely related to the relative TFP of the sector that produces good i. The impact of the capital to labor ratio on the relative price depends on the capital intensity of the production technology for the good. If the sector is more capital instensive than the manufacturing sector, then $\alpha_{it} < \alpha_m$ and the relative price increases as the capital to labor ratio increases. Note that w_t, r_t, and p_{it} are all functions of k_{mt}.

The market clearing condition for good i ($i < m$) is

$$c_{it} = Y_{it} = A_{it} K_{it}^{1-\alpha_i} L_{it}^{\alpha_i} = A_{it} k_{it}^{1-\alpha_i} L_{it}$$

or

$$p_{it}c_{it} = p_{it}A_{it}k_{it}^{1-\alpha_i}L_{it}. \tag{22}$$

From the first order condition of the firm's profit maximization problem we have

$$p_{it}A_{it}k_{it}^{1-\alpha_i} = \alpha_i^{-1}w_t = \frac{\alpha_m}{\alpha_i}A_{mt}k_{mt}^{1-\alpha_m}. \tag{23}$$

From equation (16) we have :

$$p_{it}c_{it} = \frac{\phi_i^{\varepsilon}p_{it}^{1-\varepsilon}}{\sum_{j=1}^m \phi_j^{\varepsilon}p_j^{1-\varepsilon}}\left(w_tL + r_tK_t - X_t - \sum_{j=1}^m p_{jt}b_j\right) + p_{it}b_i, \tag{24}$$

Substituting (23) and (24) into equation (22) and solving for L_{it} yields the following equation

$$L_{it} = \frac{\alpha_i}{\alpha_m}\frac{\frac{\phi_i^{\varepsilon}p_{it}^{1-\varepsilon}}{\sum_{j=1}^m \phi_j^{\varepsilon}p_j^{1-\varepsilon}}\left(w_tL + r_tK_t - X_t - \sum_{j=1}^m p_{jt}b_j\right) + p_{it}b_i}{A_{mt}k_{mt}^{1-\alpha_m}}. \tag{25}$$

Since w_t, w_t, r_t, and p_{it} are all functions of k_{mt}, L_{it} is a function of k_{mt}, K_t and X_t, for any $i < m$. Since $L_{mt} = L - \sum_{i=1}^{m-1} L_{it}$, it is also a function of k_{mt}, K_t and X_t.

Finally, the capital market clearing condition is

$$K_t = \sum_{i=1}^m K_{it}.$$

From (18) then we have

$$K_t = \sum_{i=1}^m k_{it}L_{it} = k_{mt}\sum_{i=1}^m \mu_i L_{it}. \tag{26}$$

Because L_{it} is a function of k_{mt}, K_t and X_t for all $i = 1, ..., m$. Equation (26) can be used to solve for k_{mt} as a function of K_t and X_t.

In the special case when all sectors have the same capital share, i.e., $\alpha_i = \alpha$ for all $i = 1, ..., m$. Then, $\mu_i = 1$ and equation (26) implies that

$$k_{mt} = \frac{K_t}{L}.$$

This in turn implies that

$$w_t = \alpha A_{mt} k_{mt}^{1-\alpha} = \alpha A_{mt} K_t^{1-\alpha} L^{\alpha-1}. \tag{27}$$

$$r_t = (1-\alpha) A_{mt} k_{mt}^{-\alpha} = (1-\alpha) A_{mt} K_t^{-\alpha} L^{\alpha}. \tag{28}$$

Thus,

$$Y_t = w_t L + r_t K_t = A_{mt} K_t^{1-\alpha} L^{\alpha}.$$

Equation (25) can then be written as

$$\frac{L_{it}}{L} = \frac{\phi_i^{\varepsilon} p_{it}^{1-\varepsilon}}{\sum_{j=1}^{m} \phi_j^{\varepsilon} p_j^{1-\varepsilon}} \left(1 - \frac{X_t}{Y_t} - \frac{\sum_{j=1}^{m} p_{jt} b_j}{Y_t}\right) + \frac{p_{it} b_i}{Y_t}.$$

Finally, in this special case, equation (21) simplifies to

$$p_{it} = \frac{A_{mt}}{A_{it}}$$